The Birthday Problem

The goal is to compute P(A), the probability that at least two people in the room have the same birthday. However, it is simpler to calculate P(A'), the probability that no two people in the room have the same birthday. Then, because A and A' are the only two possibilities and are also mutually exclusive, P(A) = 1 − P(A').

For example, if we have 23 people, then the probability that no two people in the room have the same birthday is:

$$P(A') = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{343}{365}$$

And the probability that two people have the same birthday is:

$$P(A) = 1 - \left(\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{343}{365}\right)$$

We can calculate this easily in a spreadsheet.

Set up these headings and numbers to begin:

	A	B	C	D	E	F
1	What is the probability that 2 people have the same Birthday?					
2	Person	Day if no pair	Possible Days	Prob	P(no pair)	P(at least 1 pair)
3	1	365	365	1	1	0

The formulas needed are:

	A	B	C	D	E	F
1	What is					
2	Person	Day if no pair	Possible Days	Prob	P(no pair)	P(at least 1 pair)
3	1	365	365	=B3/C3	=D3	=1-E3
4	=A3+1	=B3-1	365	=B4/C4	=E3*D4	=1-E4
5	=A4+1	=B4-1	365	=B5/C5	=E4*D5	=1-E5
6	=A5+1	=B5-1	365	=B6/C6	=E5*D6	=1-E6

Verify that your formulas produce these numbers:

	A	B	C	D	E	F
1	What is the probability that 2 people have the same Birthday?					
2	Person	Day if no pair	Possible Days	Prob	P(no pair)	P(at least 1 pair)
3	1	365	365	1	1	0
4	2	364	365	0.99726	0.997260274	0.002739726
5	3	363	365	0.994521	0.991795834	0.008204166
6	4	362	365	0.991781	0.983644088	0.016355912
7	5	361	365	0.989041	0.972864426	0.027135574
8	6	360	365	0.986301	0.959537516	0.040462484

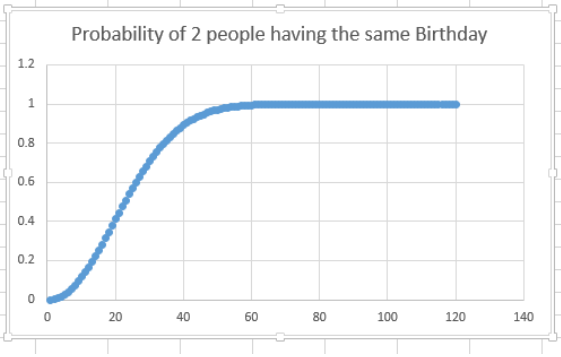
Fill down so that the first column is about 120.

Notice that the probability that you will have 2 people with the same birthday in a room of 23 is 50%.

We have 31 students, the probability that two people have the same birthday is 73%.

Graph the probabilities with a scatterplot:

	A	B	C	D	E	F
1	What is the probability that 2 people have the same Birthday?					
2	Person	Day if no pair	Possible Days	Prob	P(no pair)	P(at least 1 pair)
3	1	365	365	1	1	0
4	2	364	365	0.99726	0.997260274	0.002739726
5	3	363	365	0.994521	0.991795834	0.008204166
6	4	362	365	0.991781	0.983644088	0.016355912
7	5	361	365	0.989041	0.972864426	0.027135574
8	6	360	365	0.986301	0.959537516	0.040462484
9	7	359	365	0.983562	0.943764297	0.056235703
10	8	358	365	0.980822	0.925664708	0.074335292
11	9	357	365	0.978082	0.905376166	0.094623834
12	10	356	365	0.975342	0.883051822	0.116948178
13	11	355	365	0.972603	0.858858622	0.141141378
14	12	354	365	0.969863	0.832975211	0.167024789
15	13	353	365	0.967123	0.805589725	0.194410275
16	14	352	365	0.964384	0.776897488	0.223102512
17	15	351	365	0.961644	0.74709868	0.25290132
18	16	350	365	0.958904	0.716395995	0.283604005



Show Ms. Gorski your spreadsheet when you are done.