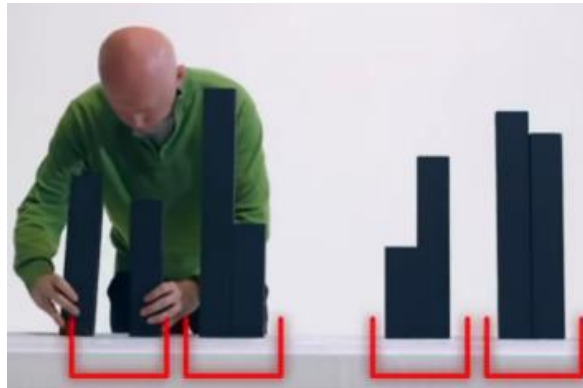
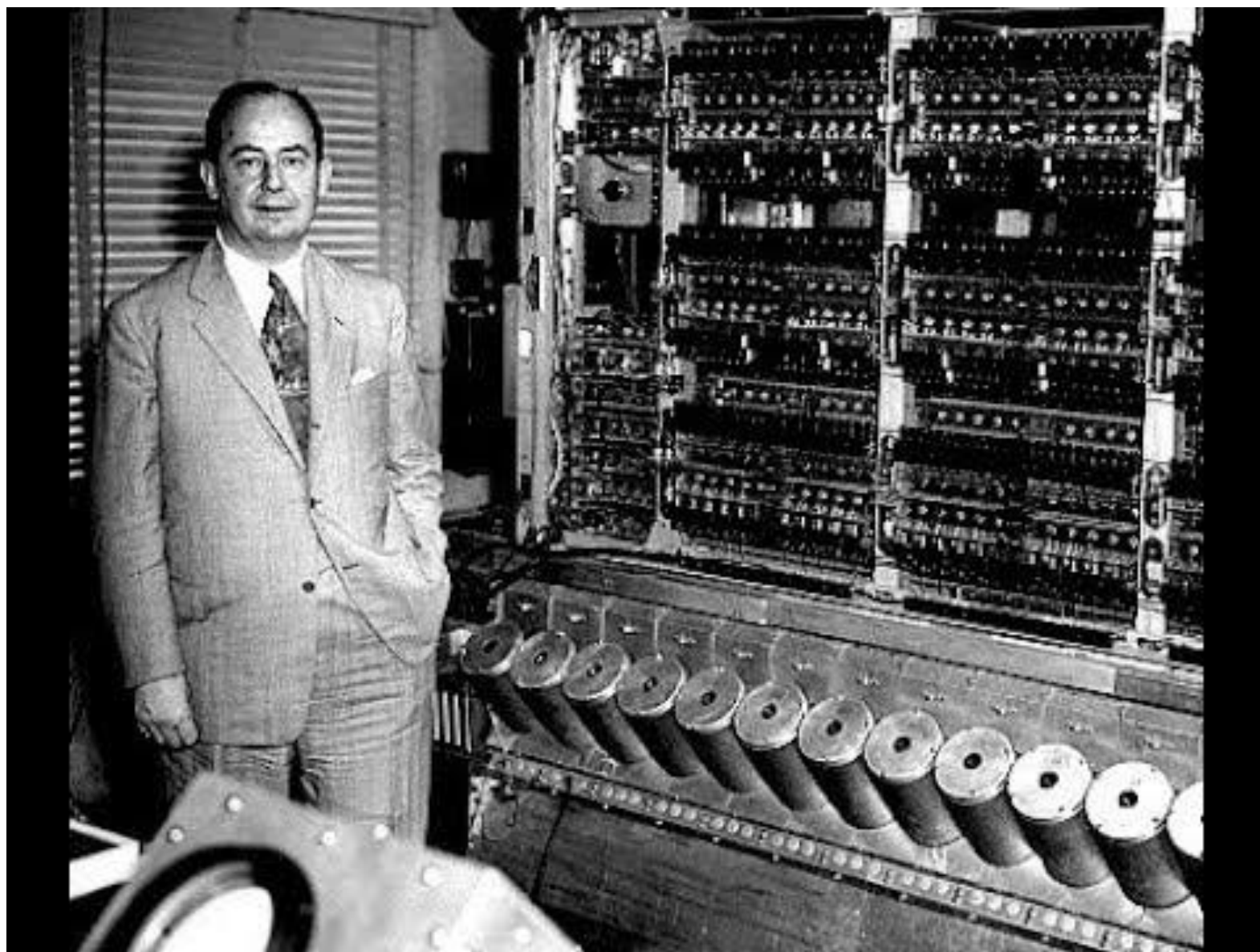


Mergesort

Recursive and $O(n \log n)$

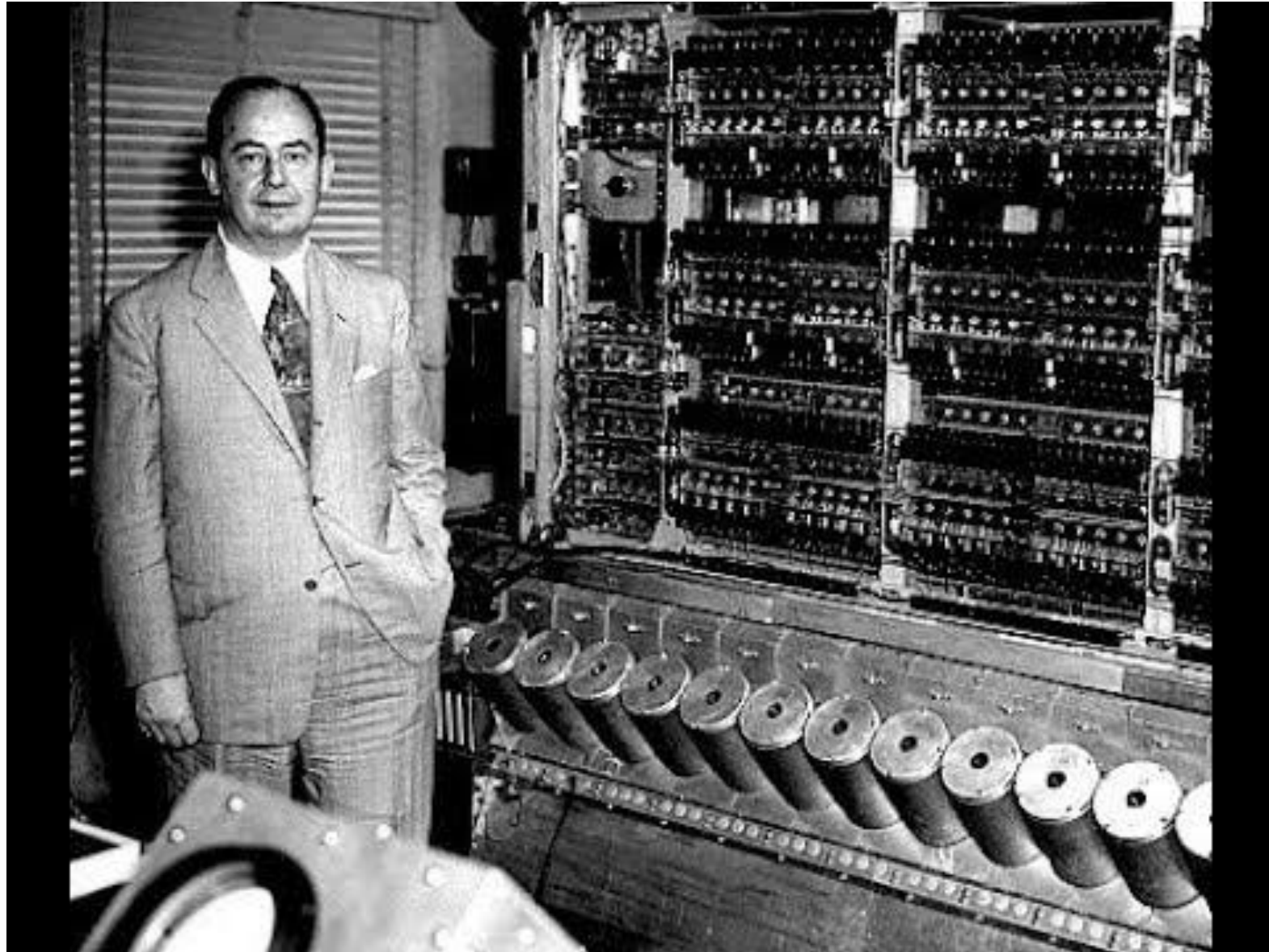


John Von Neumann



John Von Neumann

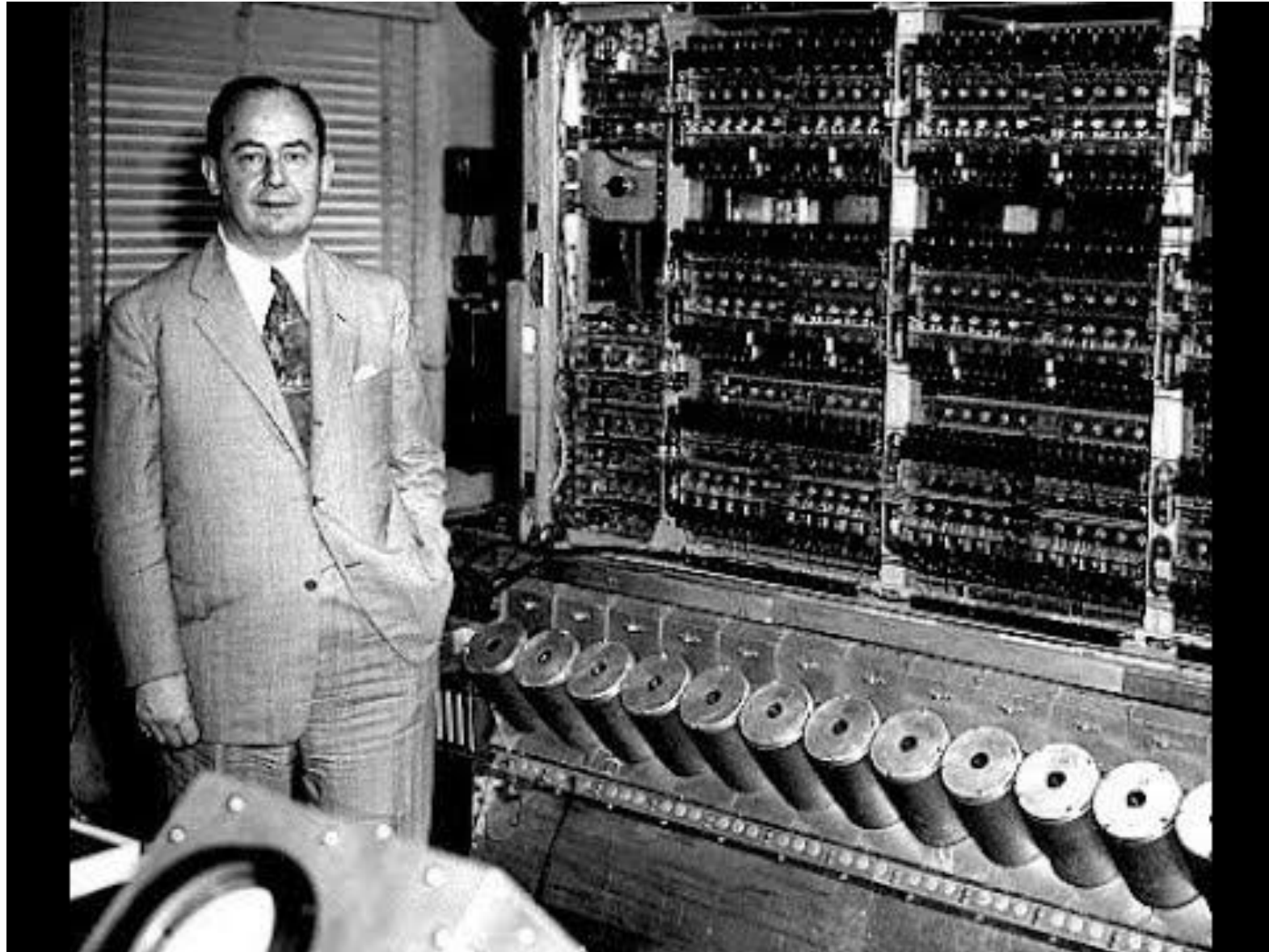
Foundations of
Mathematics



John Von Neumann

Game
theory

Foundations of
Mathematics

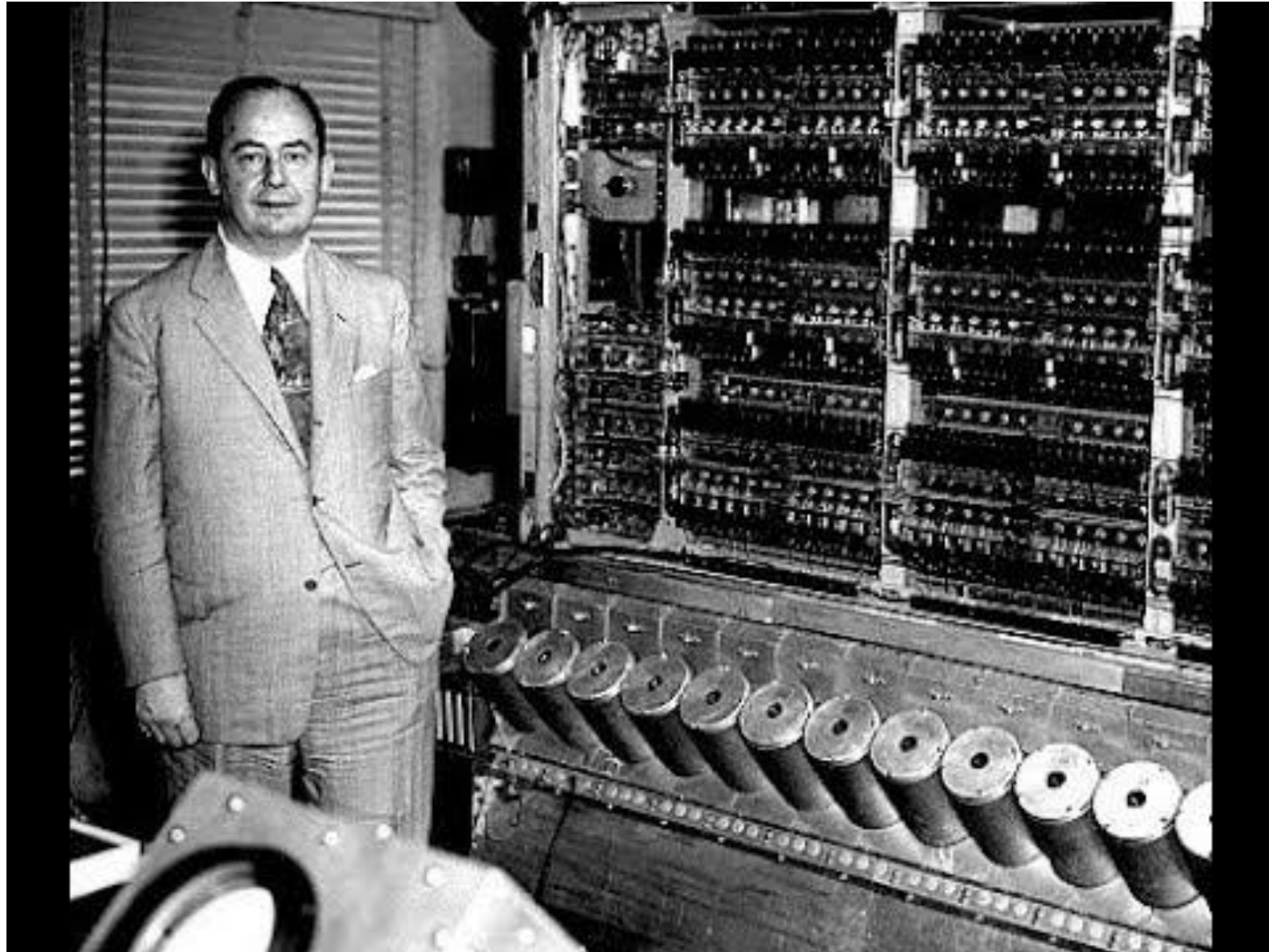


John Von Neumann

Game
theory

Foundations of
Mathematics

Digital
Computer



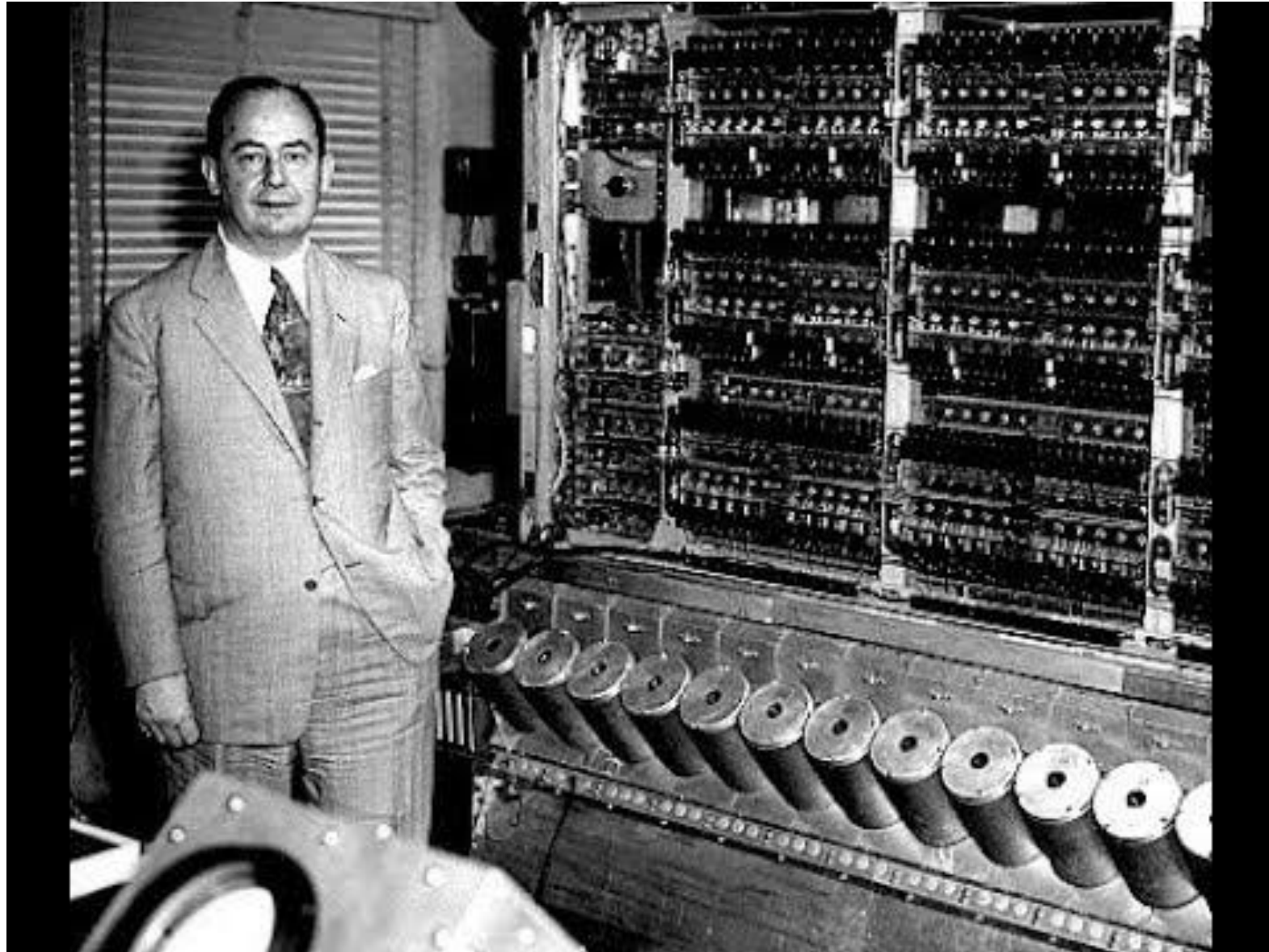
John Von Neumann

Game
theory

Foundations of
Mathematics

Digital
Computer

ENIAC



John Von Neumann

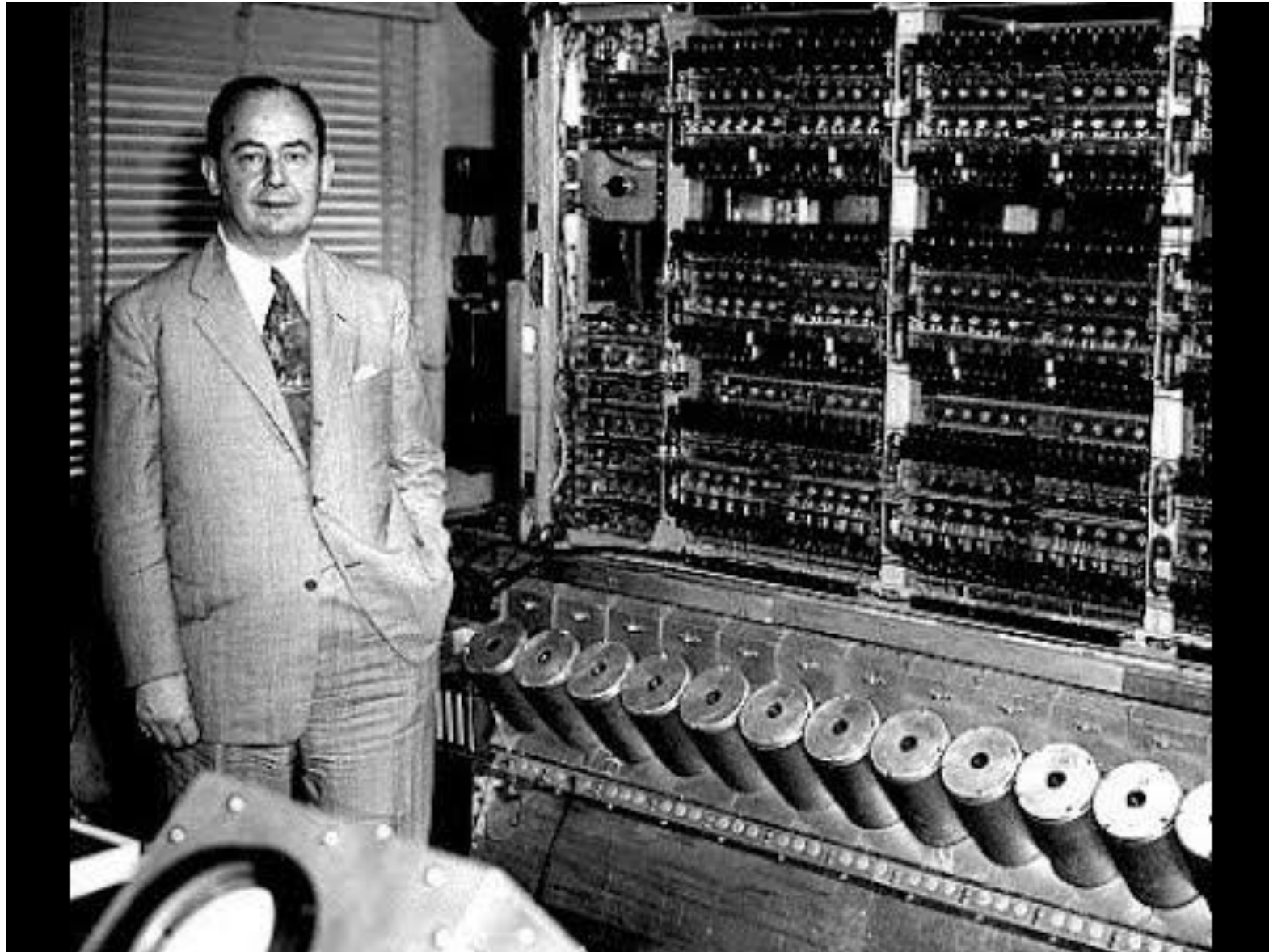
Game
theory

Foundations of
Mathematics

Digital
Computer

ENIAC

First Climate
Modelling
Software



John Von Neumann

Game
theory

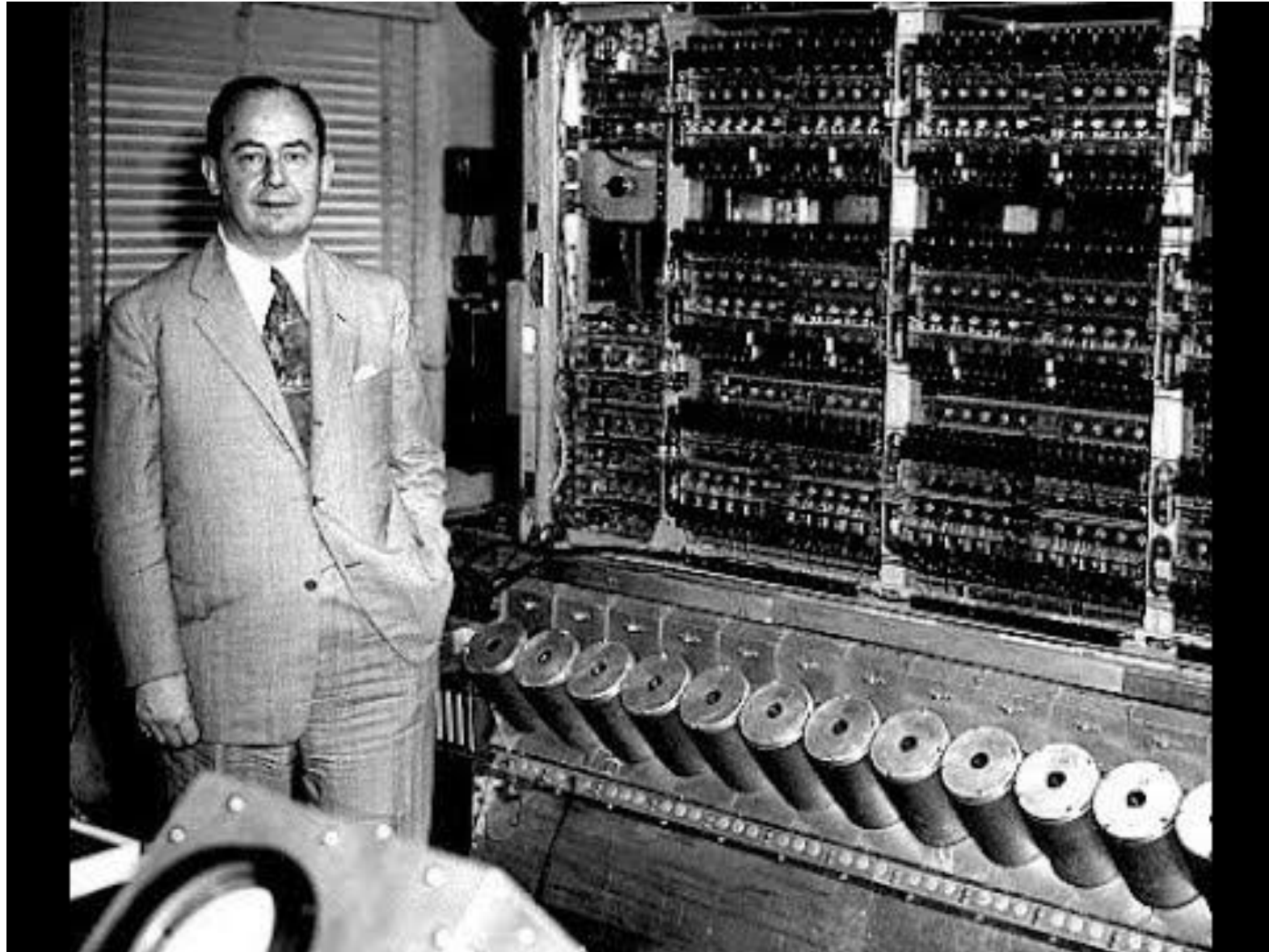
Foundations of
Mathematics

Digital
Computer

ENIAC

First Climate
Modelling
Software

Quantum
Mechanics



John Von Neumann

Game
theory

Manhattan
Project

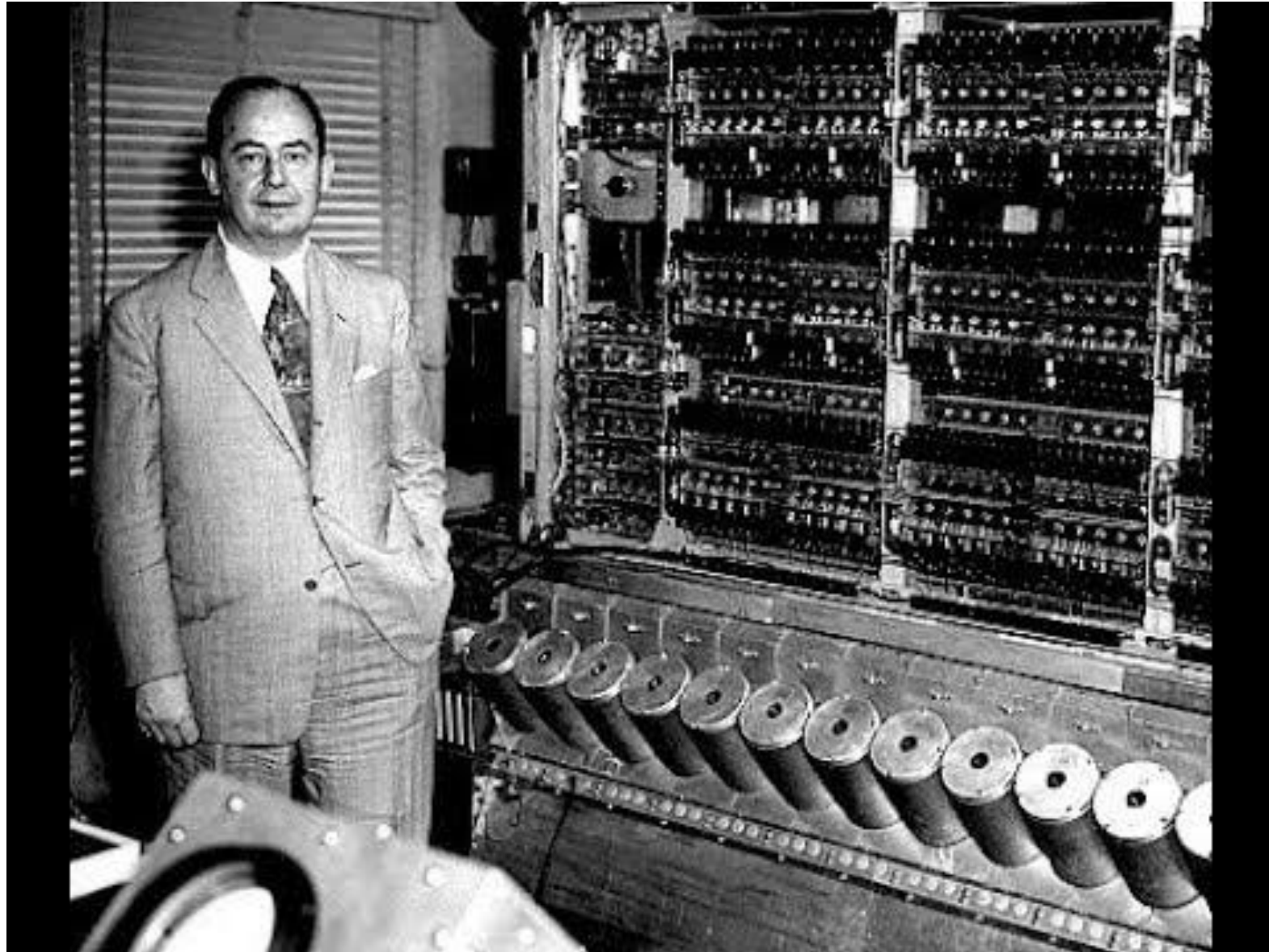
Foundations of
Mathematics

Digital
Computer

ENIAC

First Climate
Modelling
Software

Quantum
Mechanics





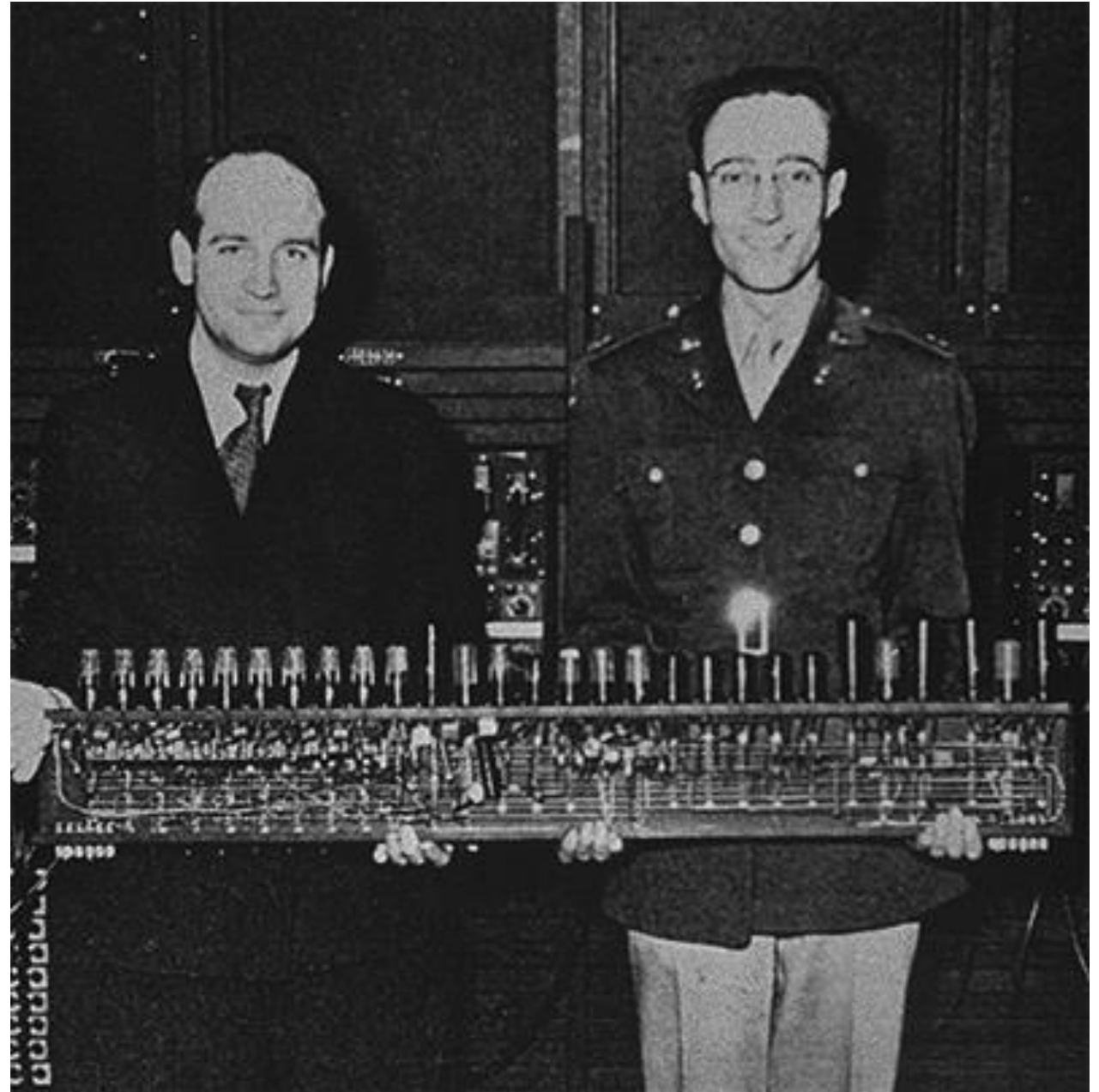
“Keeping up with him was ... impossible. The feeling was you were on a tricycle chasing a racing car.”



To gain a measure of von Neumann's achievements, consider that had he lived a normal span of years, he would certainly have been a recipient of a **Nobel Prize** in **economics**. And if there were Nobel Prizes in **computer science** and **mathematics**, he would have been honored by these, too. [In addition, von Neumann should be] thought of as a triple Nobel laureate or, possibly, a 4-fold winner, for his work in **physics**, in particular, quantum mechanics.

John von Neumann
planned out this
algorithm in 1945.

Programming didn't
yet exist.



Merge Means?



Merge Means?



To join two
things
together

Merge Means?

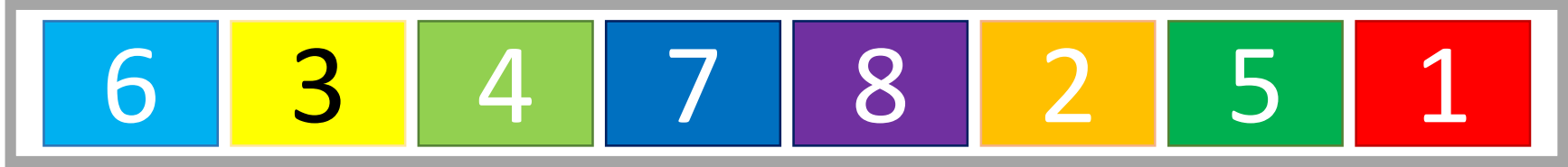


In an orderly
fashion

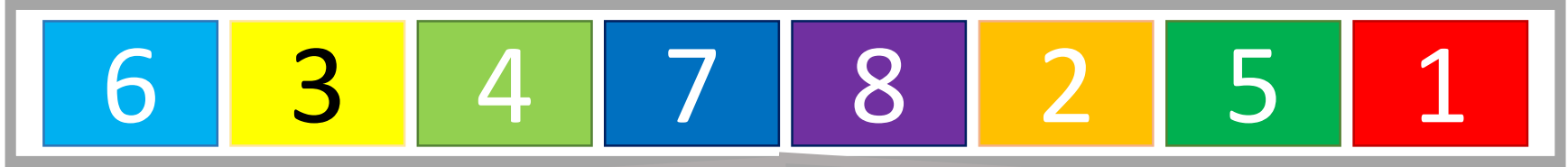
To join two
things
together

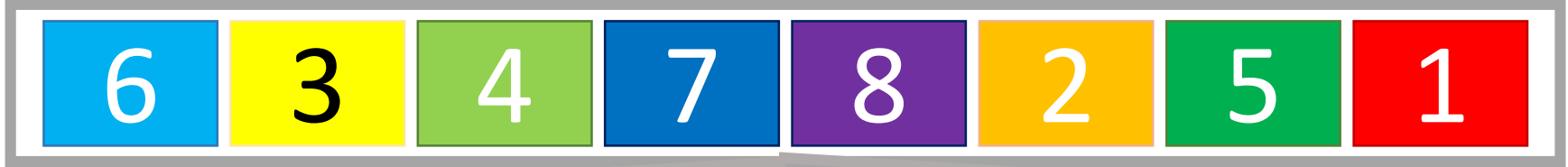
Conceptually, merge sort works as follows:

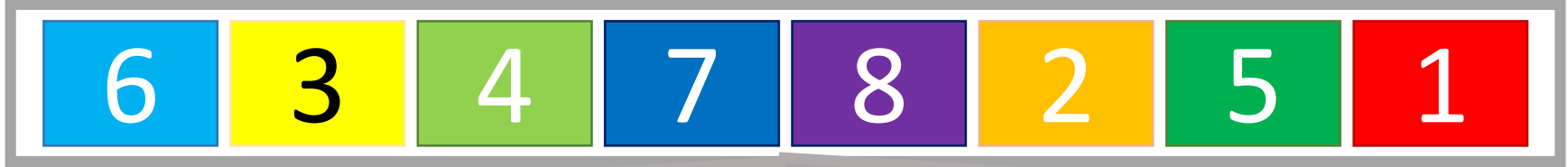
- **Divide** the unsorted list into n sublists, each containing 1 element (a list of 1 element is considered sorted).
- Repeatedly **merge** sublists to produce new sorted sublists until there is only 1 sublist remaining. This will be the sorted list.

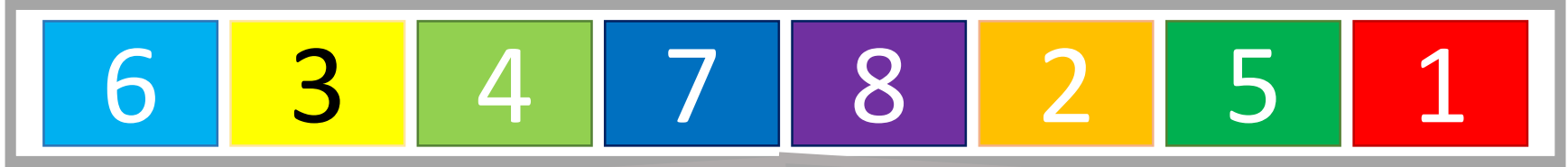


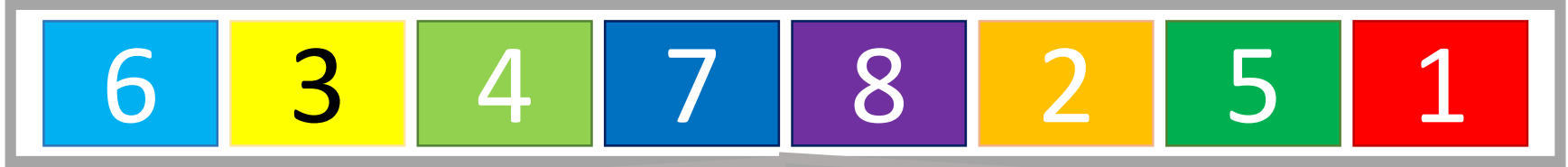


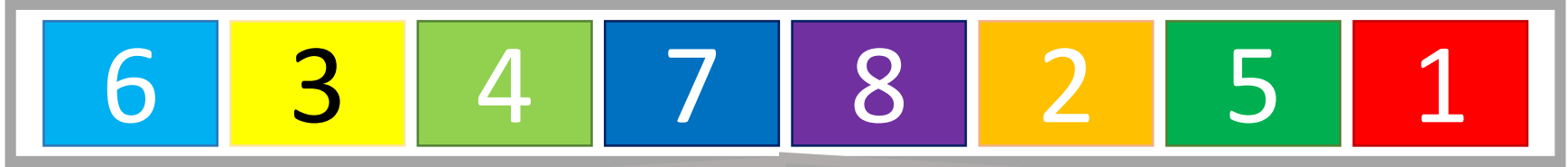


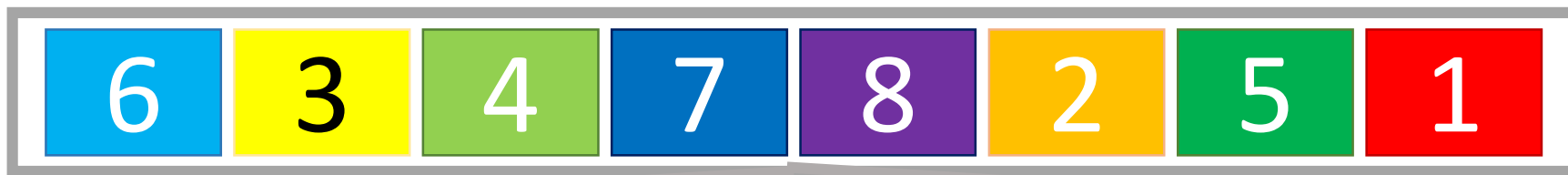


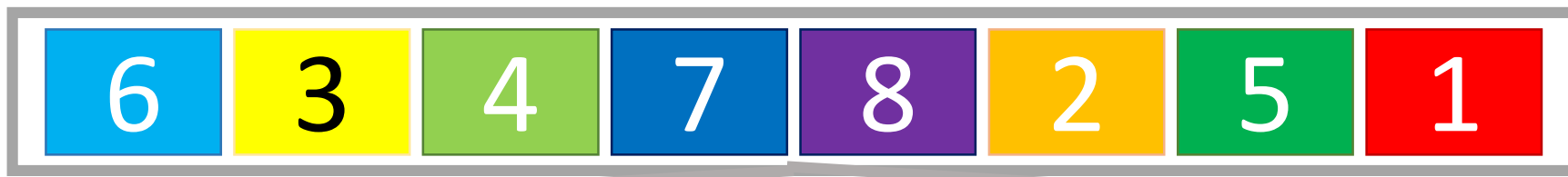


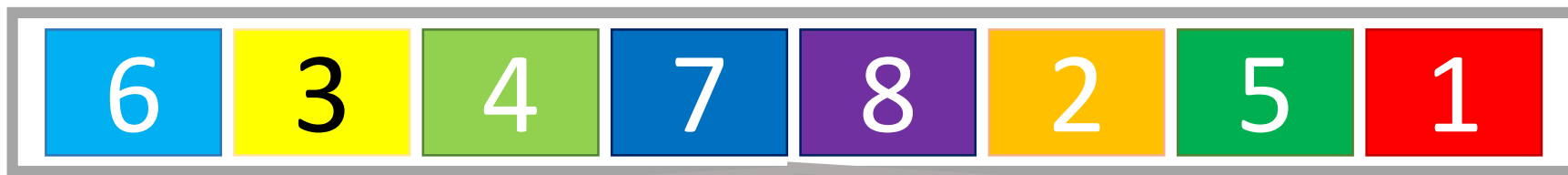


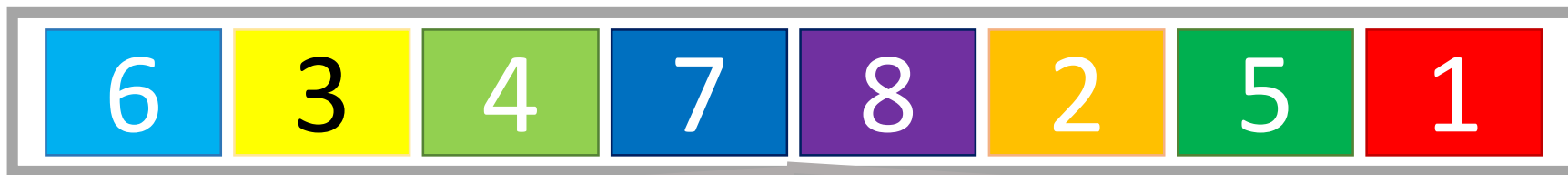




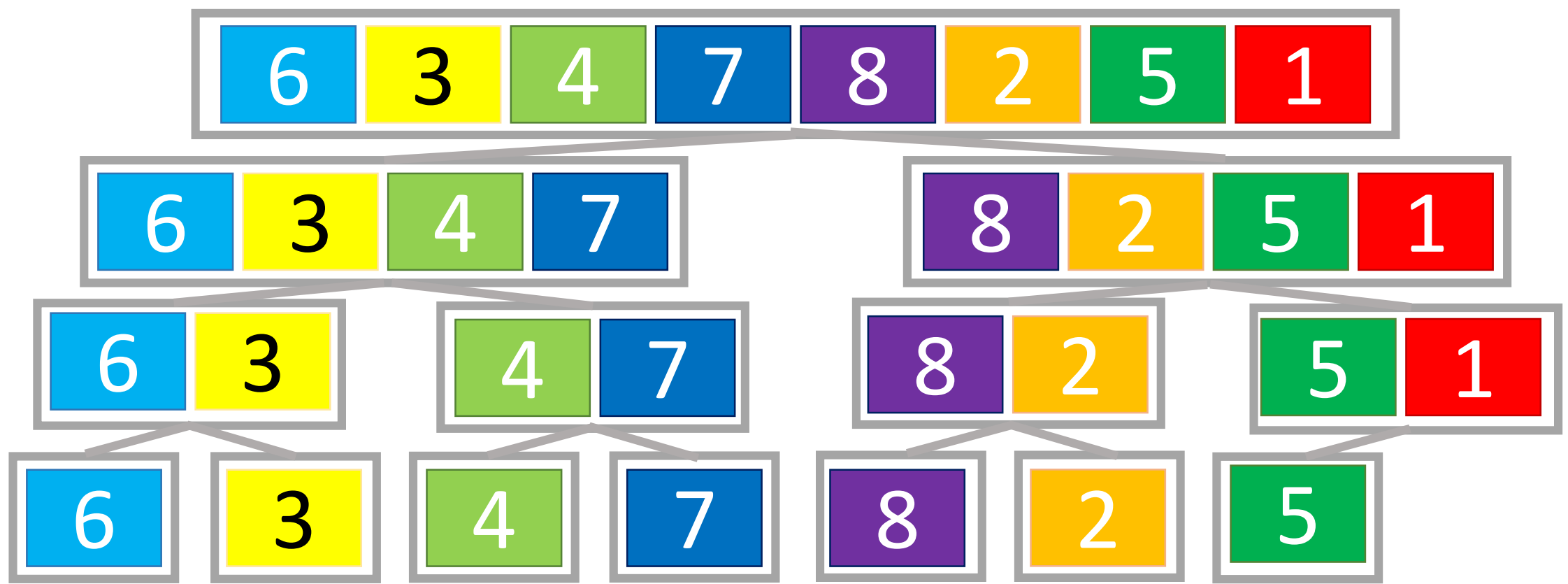


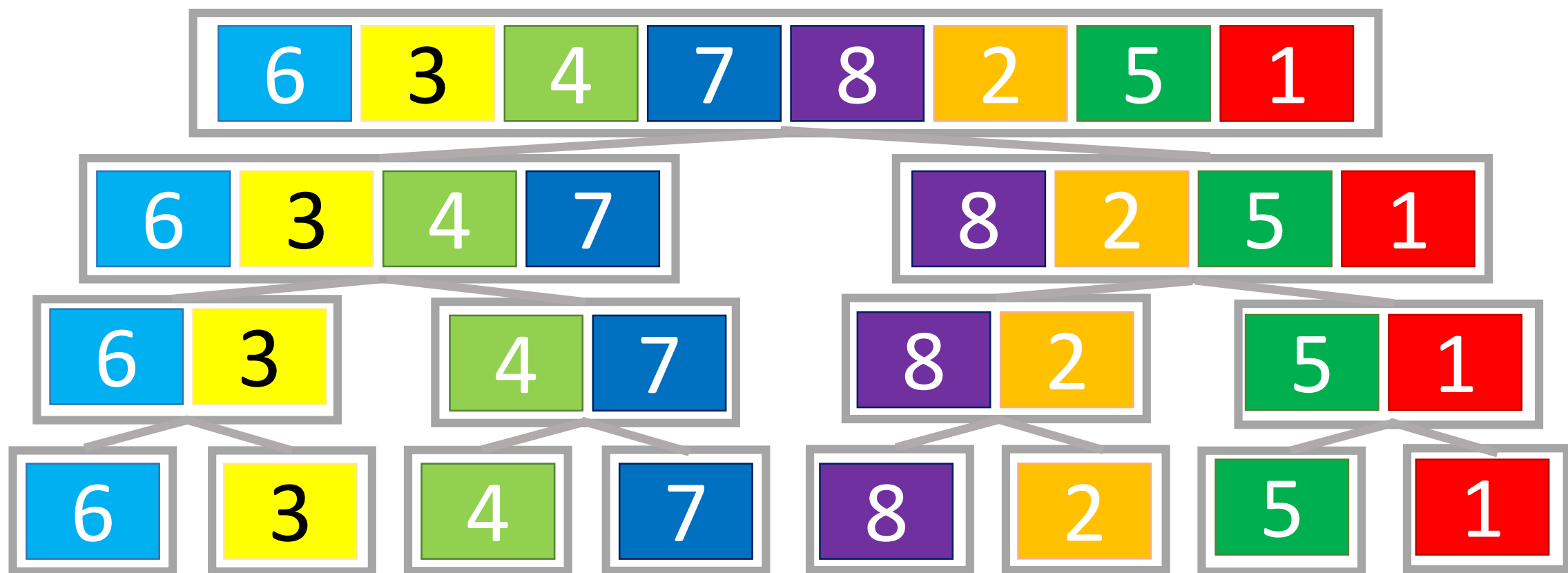


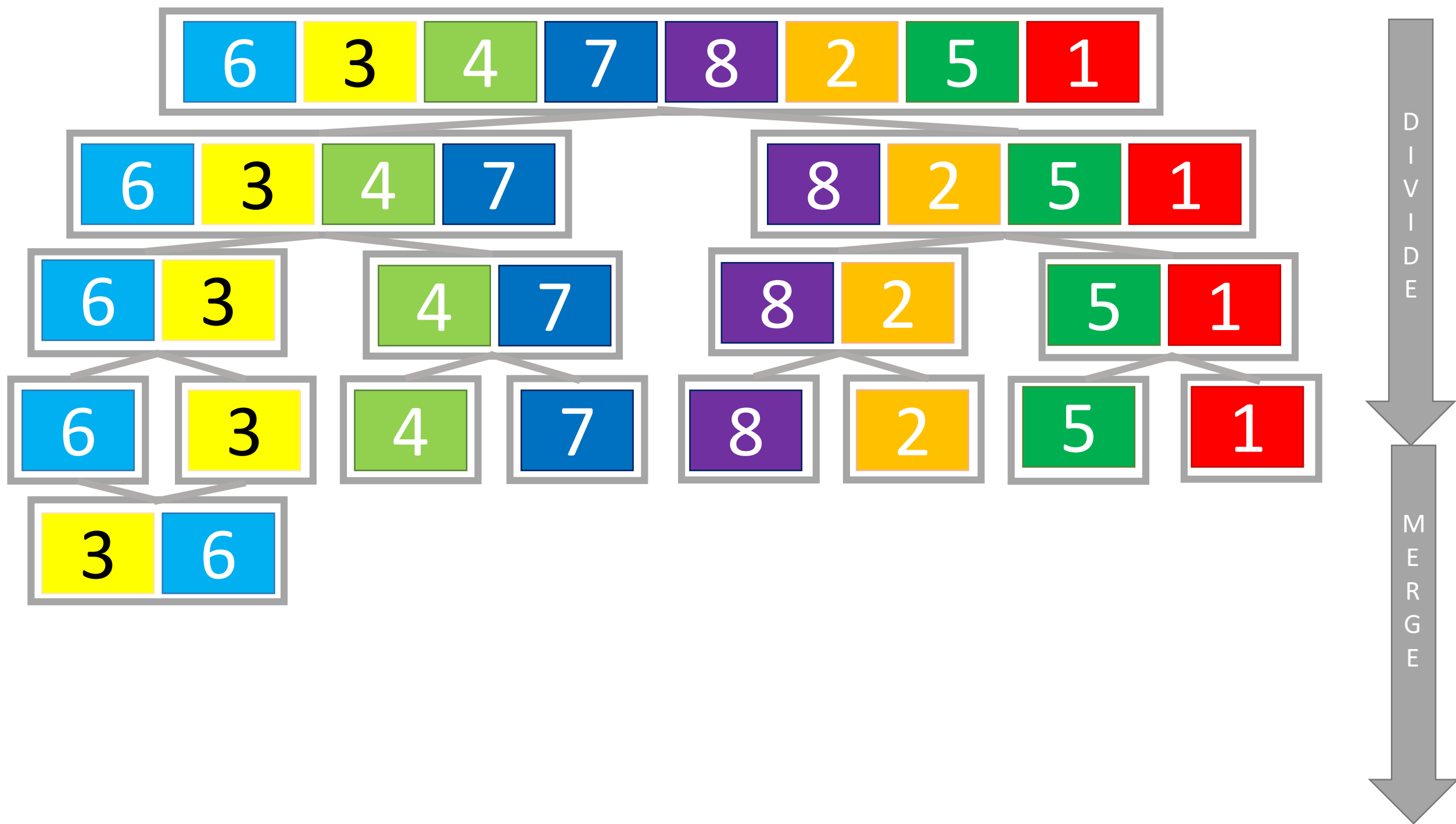


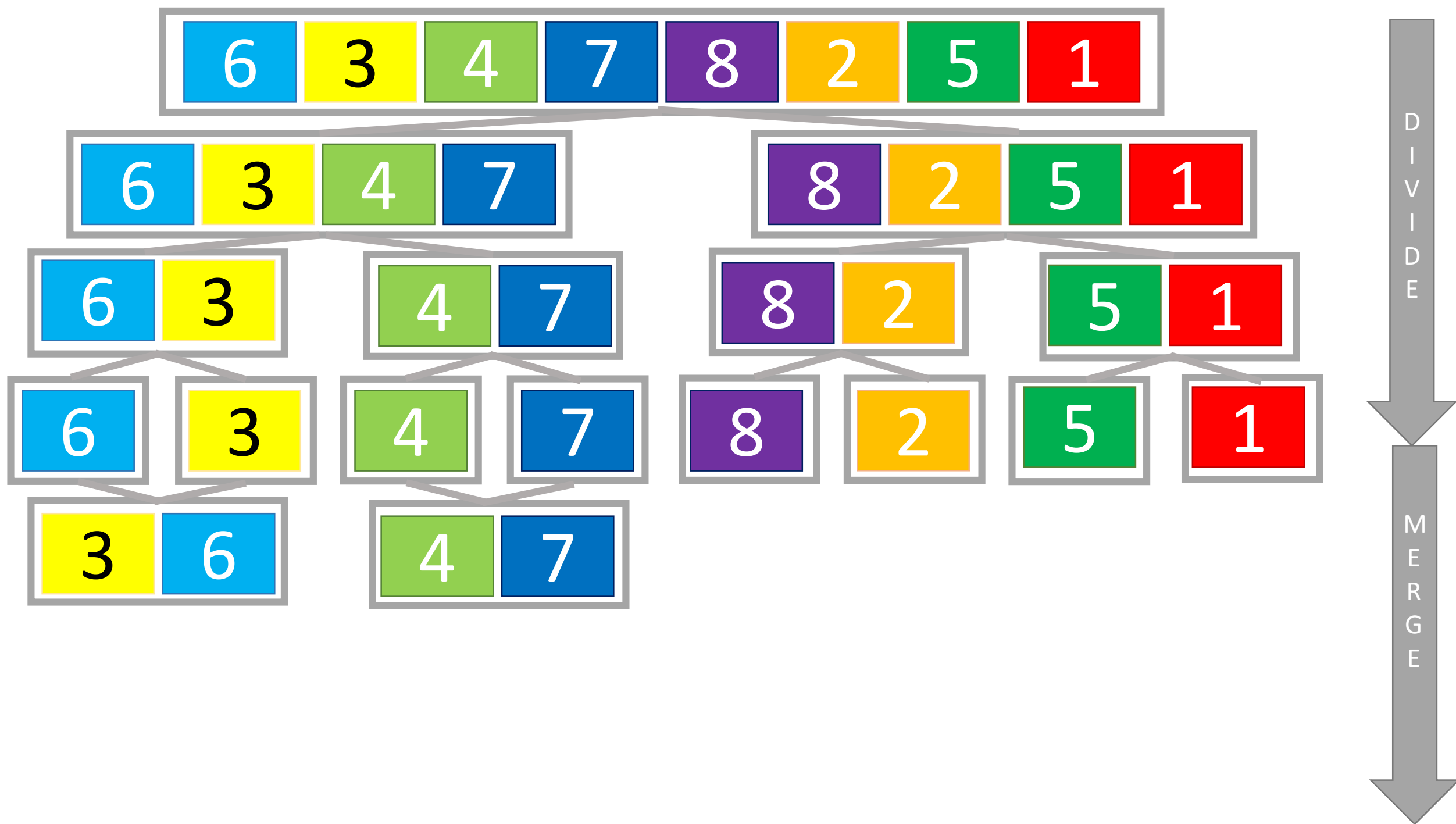


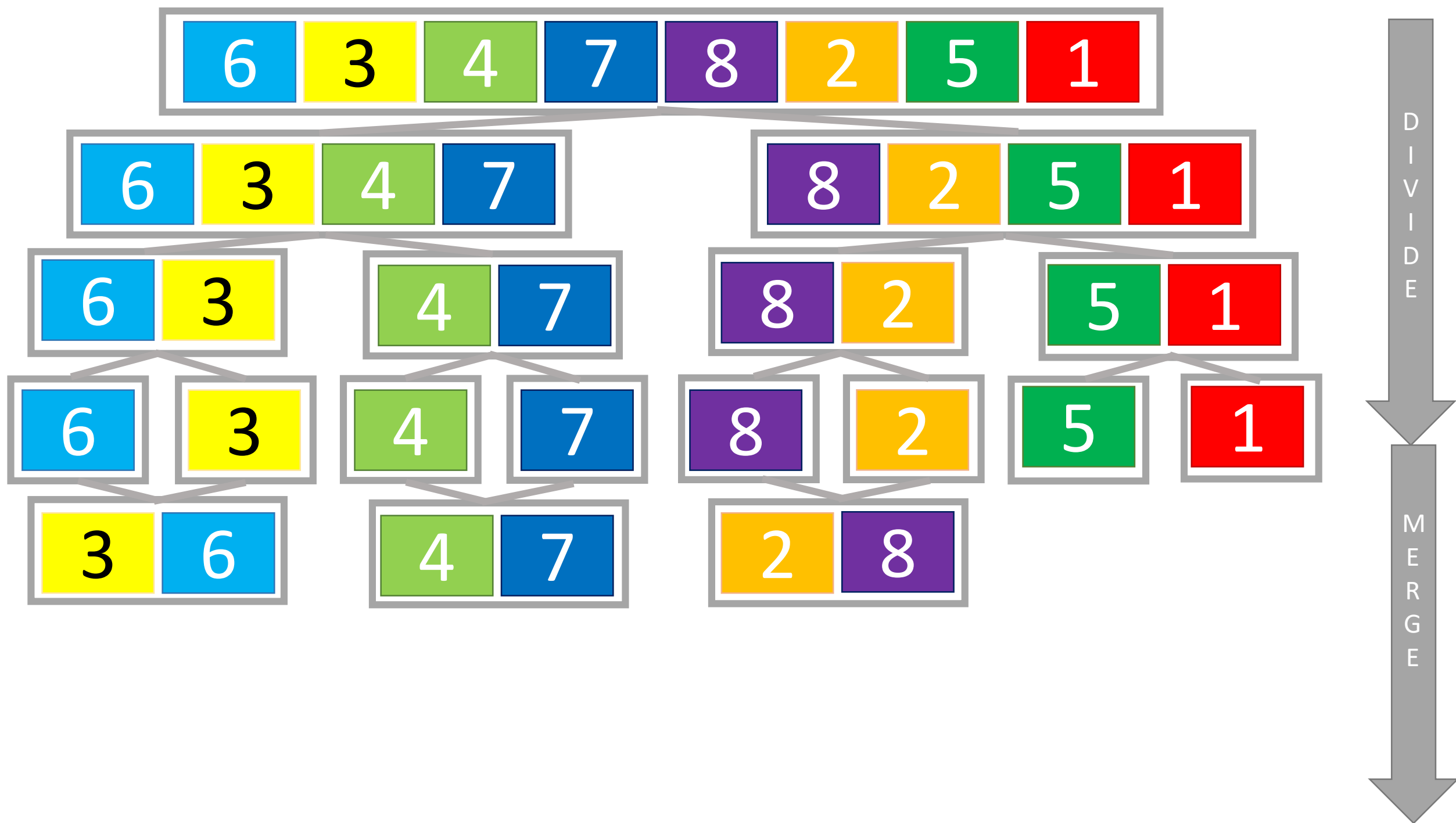


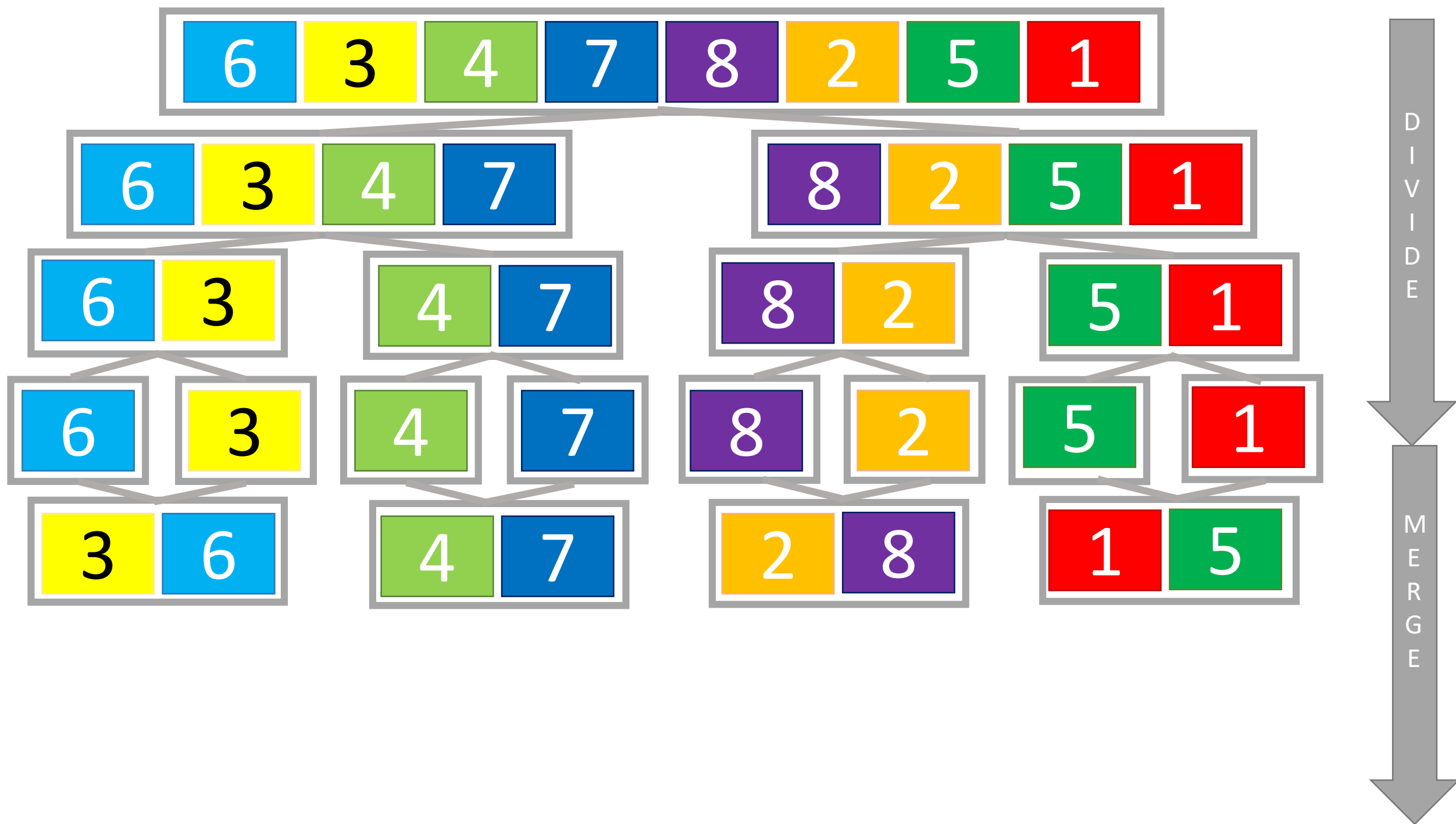


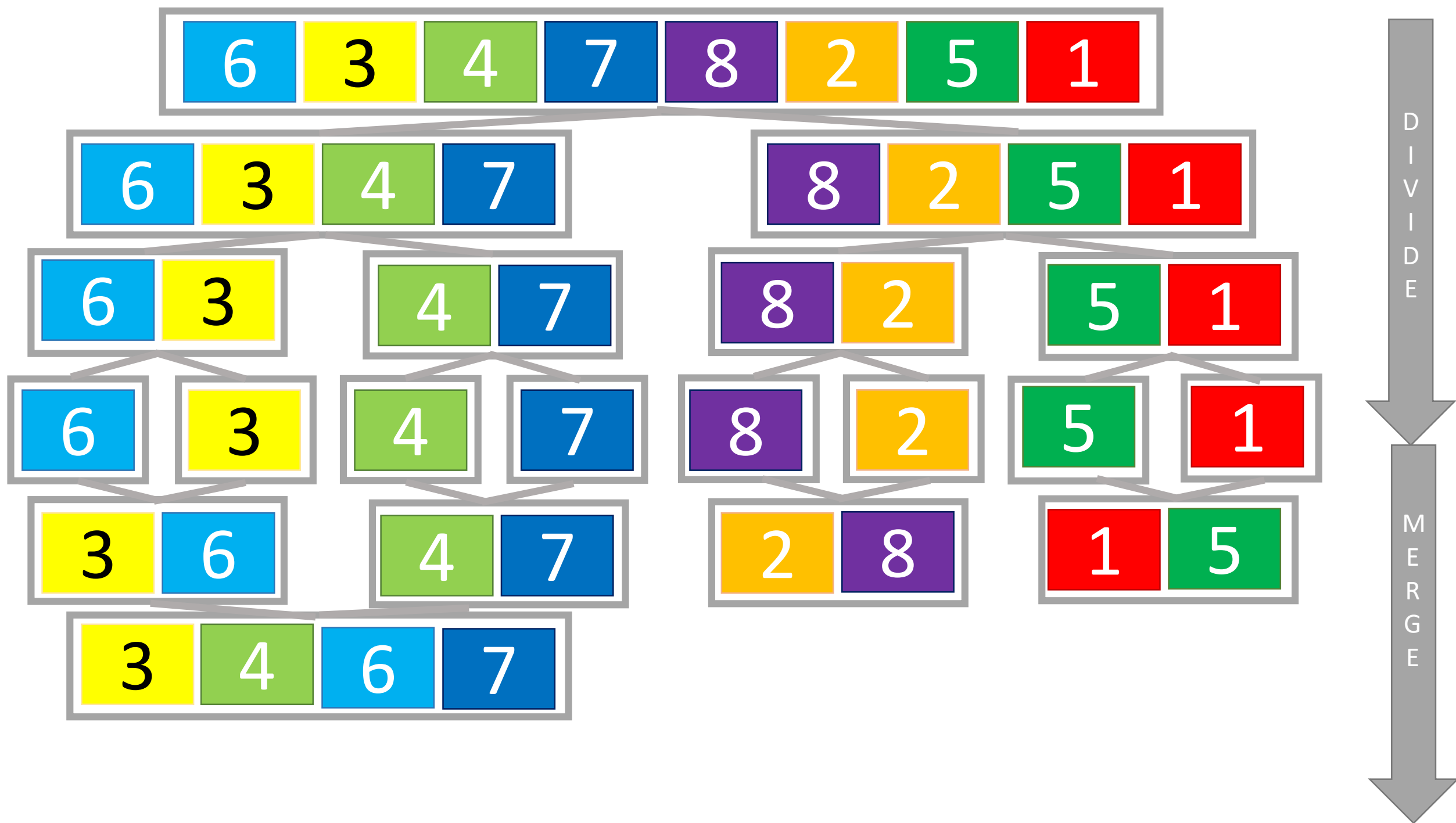


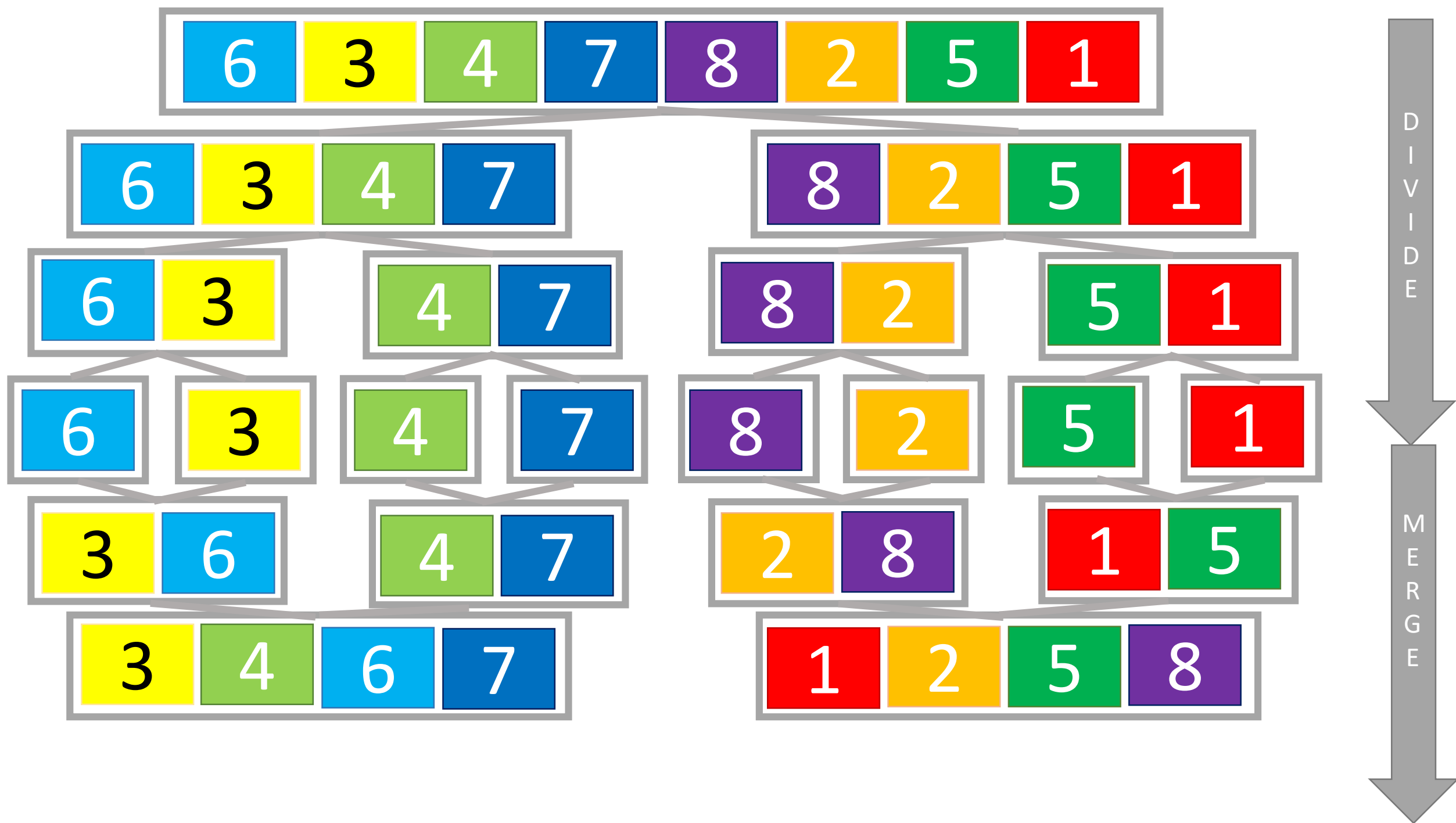


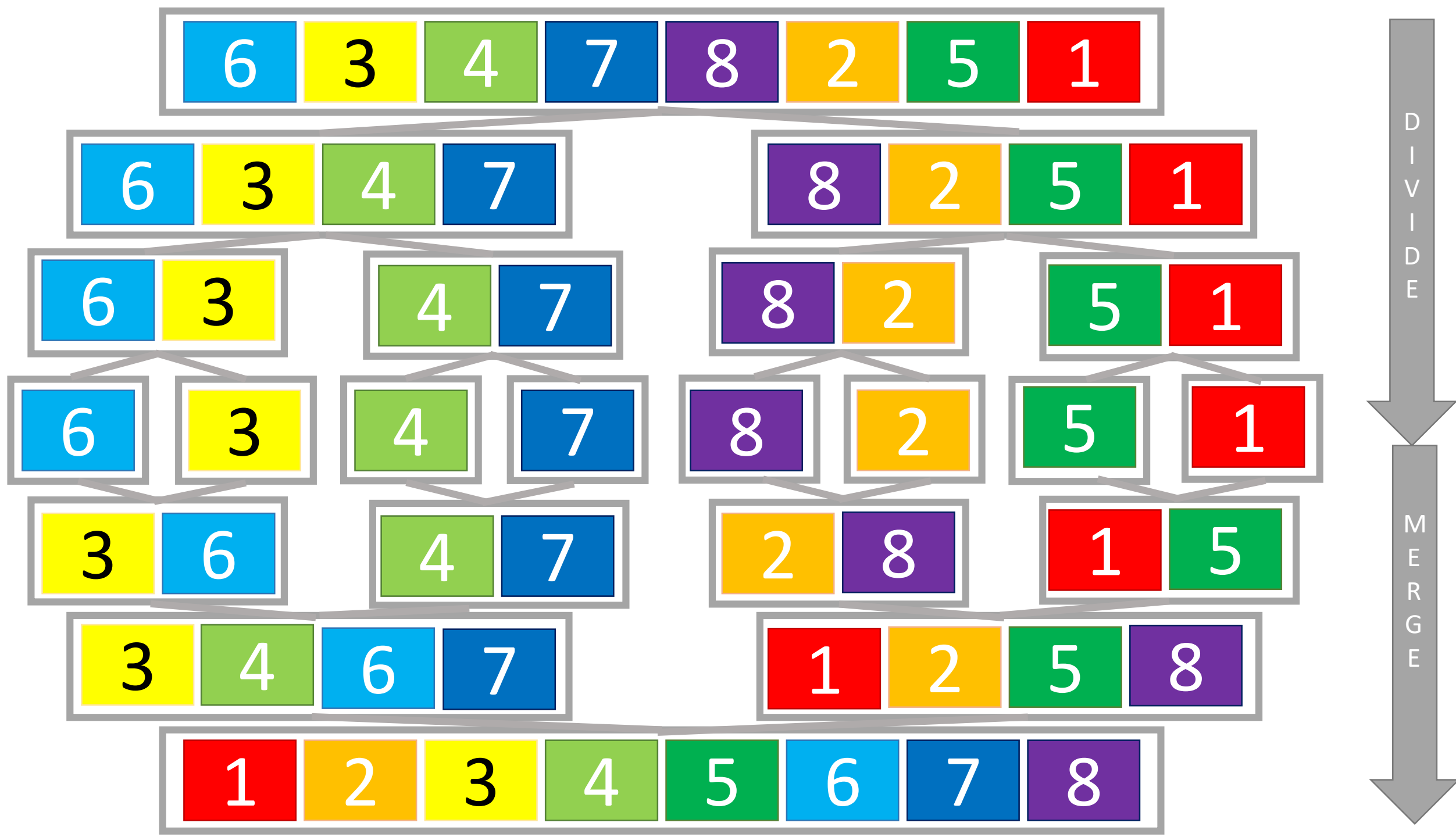






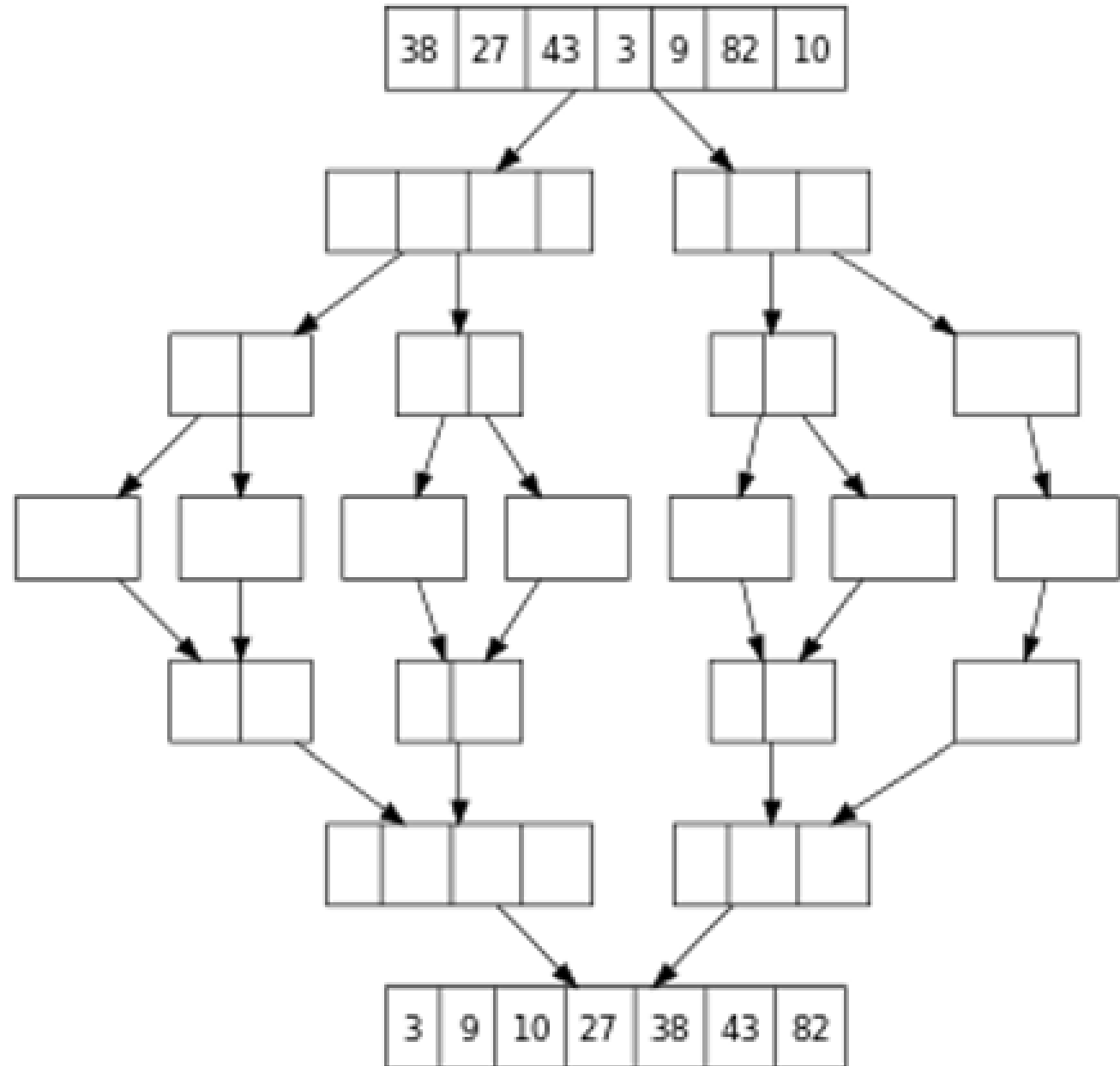






How do you
handle uneven
array sizes?

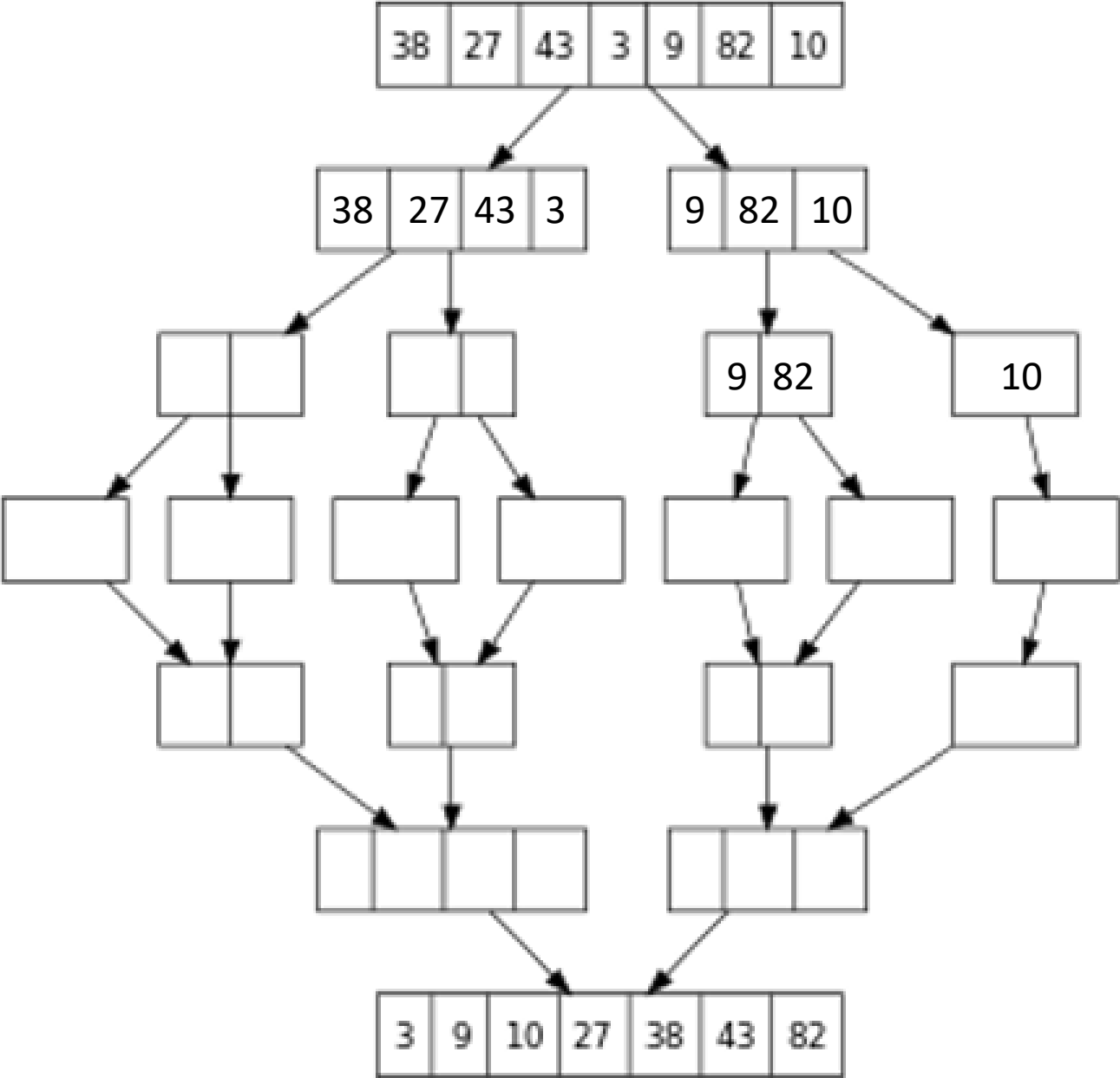
0. Fill in both tracings of mergesort:



0. Fill in both tracings of mergesort:

How do you handle uneven array sizes?

Follow the path given.



The code:

```
public int[] mergeSort(int[] array) {  
    if (array.length <= 1)  
        return array;  
  
    else {  
        int middle = array.length / 2;  
        int firstHalf = mergeSort(array[0..middle - 1]);  
        int secondHalf = mergeSort(array[middle..array.length - 1]);  
        return merge(firstHalf, secondHalf);  
    }  
}
```

The code:

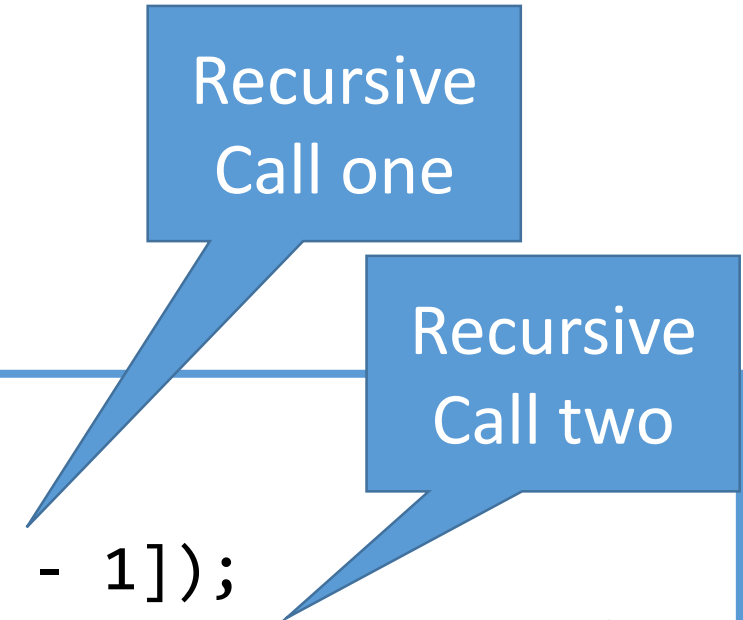
```
public int[] mergeSort(int[] array) {  
    if (array.length <= 1)  
        return array;  
  
    else {  
        int middle = array.length / 2;  
        int firstHalf = mergeSort(array[0..middle - 1]);  
        int secondHalf = mergeSort(array[middle..array.length - 1]);  
        return merge(firstHalf, secondHalf);  
    }  
}
```

Recursive
Call one



The code:

```
public int[] mergeSort(int[] array) {  
    if (array.length <= 1)  
        return array;  
  
    else {  
        int middle = array.length / 2;  
        int firstHalf = mergeSort(array[0..middle - 1]);  
        int secondHalf = mergeSort(array[middle..array.length - 1]);  
        return merge(firstHalf, secondHalf);  
    }  
}
```



The diagram consists of two blue rectangular boxes with white text. The first box, labeled 'Recursive Call one', has a blue arrow pointing to the line `int firstHalf = mergeSort(array[0..middle - 1]);` in the code. The second box, labeled 'Recursive Call two', has a blue arrow pointing to the line `int secondHalf = mergeSort(array[middle..array.length - 1]);` in the code.

The code:

```
public int[] mergeSort(int[] array) {
```

```
    if (array.length <= 1)  
        return array;
```

```
    else {  
        int middle = array.length / 2;  
        int firstHalf = mergeSort(array[0..middle - 1]);  
        int secondHalf = mergeSort(array[middle..array.length - 1]);  
        return merge(firstHalf, secondHalf);  
    }
```

```
}
```

Recursive
Call one

Recursive
Call two

Merge the
two sorted
arrays

A reminder of a previous card:

Algorithm Speeds

- $O(1)$ – swap two values, size of a array
- $O(\log n)$ – binary search
- $O(n)$ – max, min, average, print, linear search
- $O(n \log n)$ – **merge sort**, quick sort
- $O(n^2)$ – bubble sort, selection sort
- $O(n!)$ – Bogo sort

Merge sort's speed

$O(n \log n)$

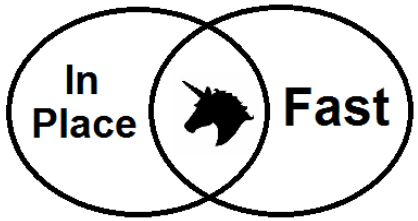
Merge sort's speed

$O(n \log n)$

Merge,
one loop

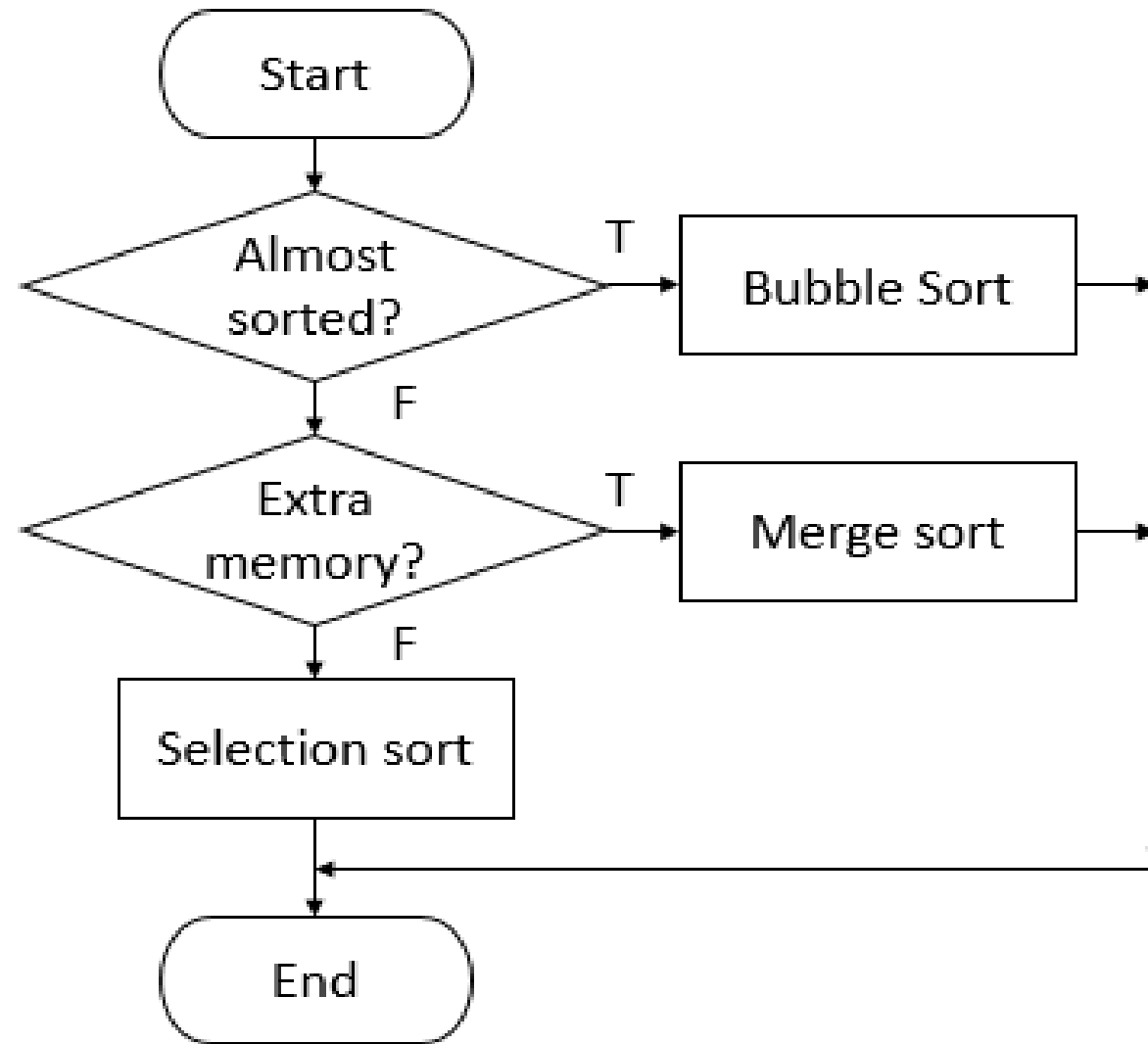
Division,
recursive

Mergesort Characteristics



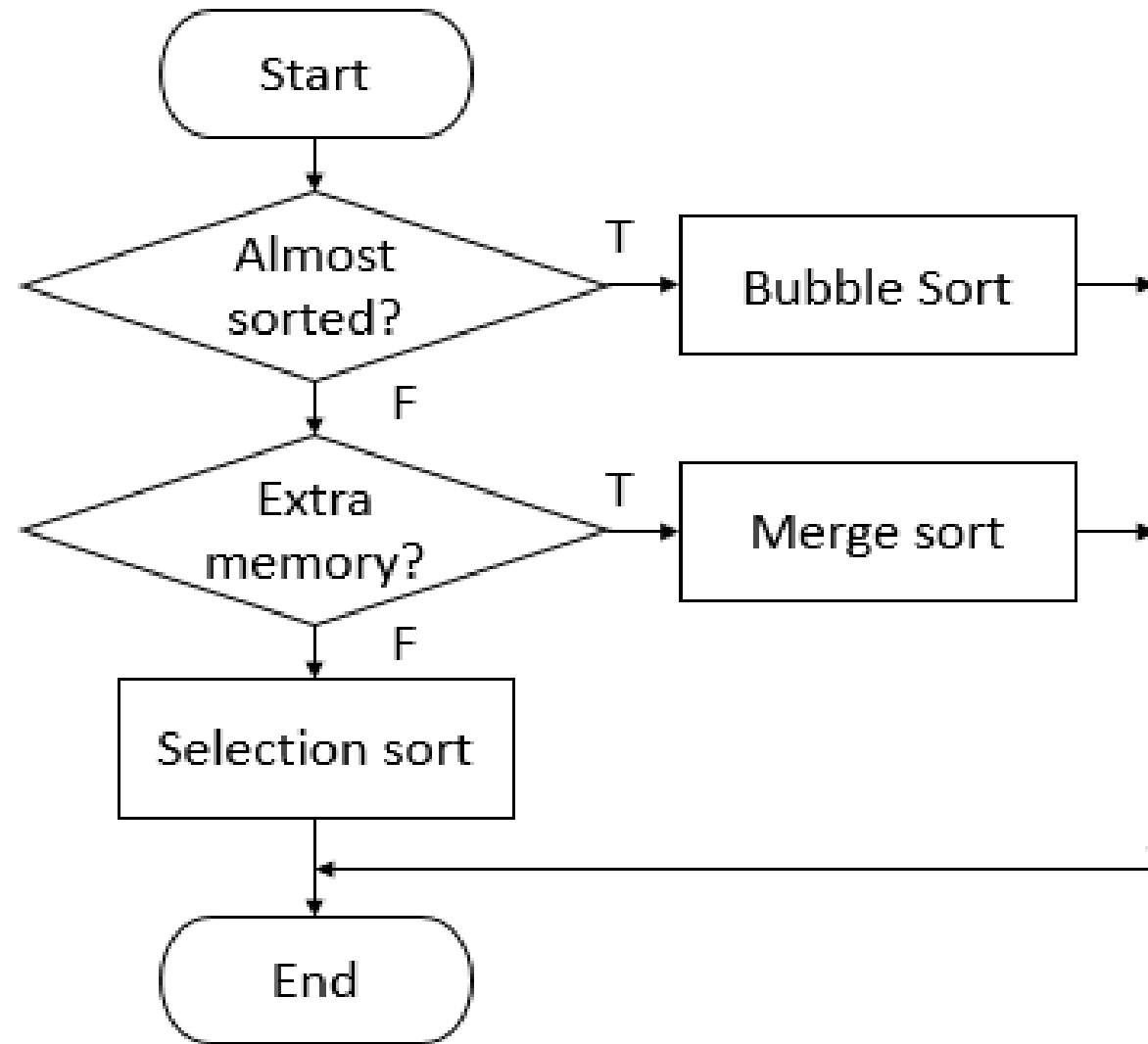
- **Algorithm:** First, divide the array recursively until you reach the base case: an array of one element. Second, repeatedly merge the array together.
- **Speed:** $n \log n$. Slightly slower than Quicksort.
- Mergesort is **not an in-place** algorithm. It does not use swaps to move elements around. Additional memory is needed for the recursive calls.
- **Trade-off:** Mergesort gains its incredible speed by using extra memory.

How do you
choose which
algorithm to
use?



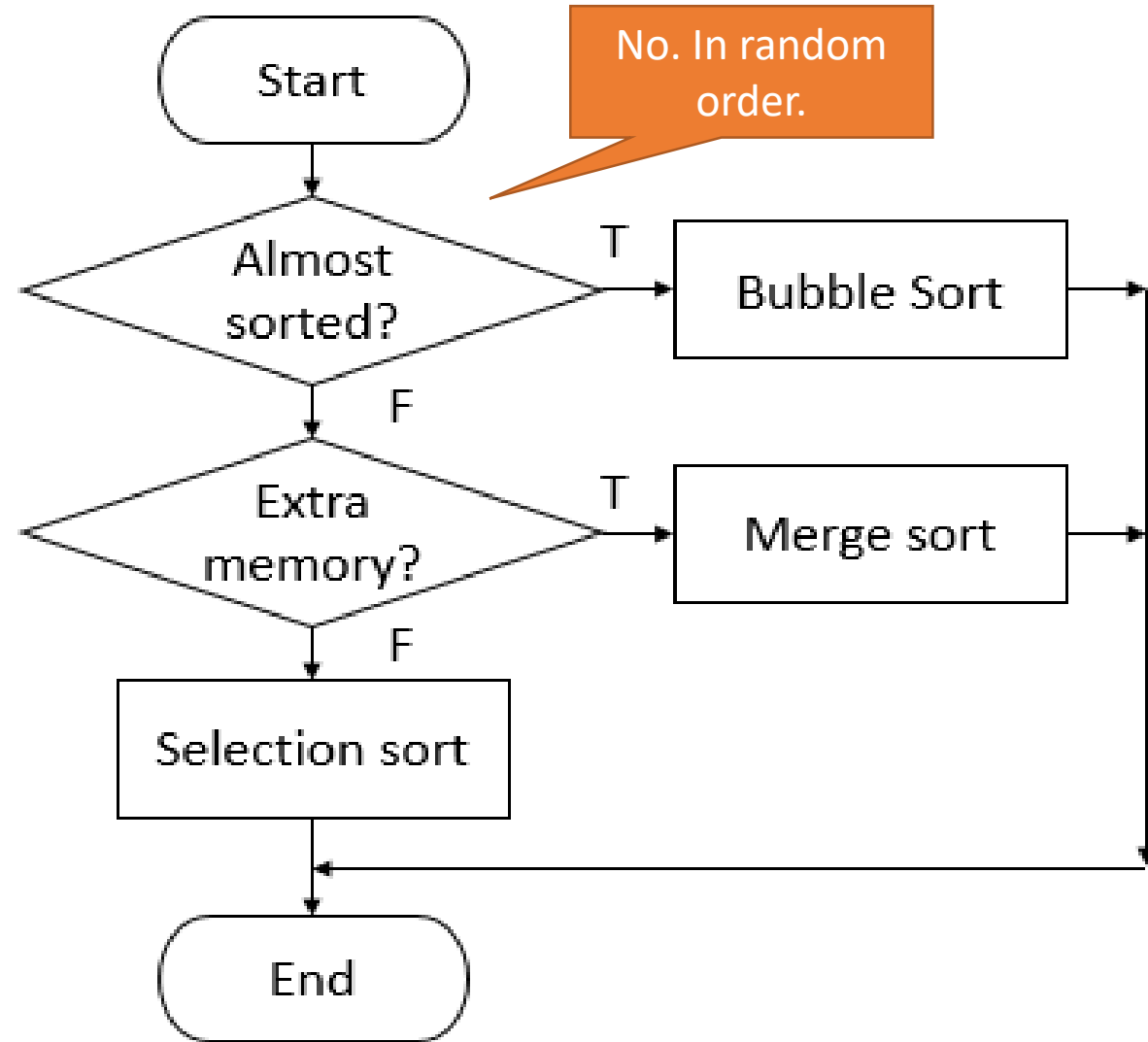
How do you choose which algorithm to use?

The array is 200 elements long (not very big), and it is in completely random order.



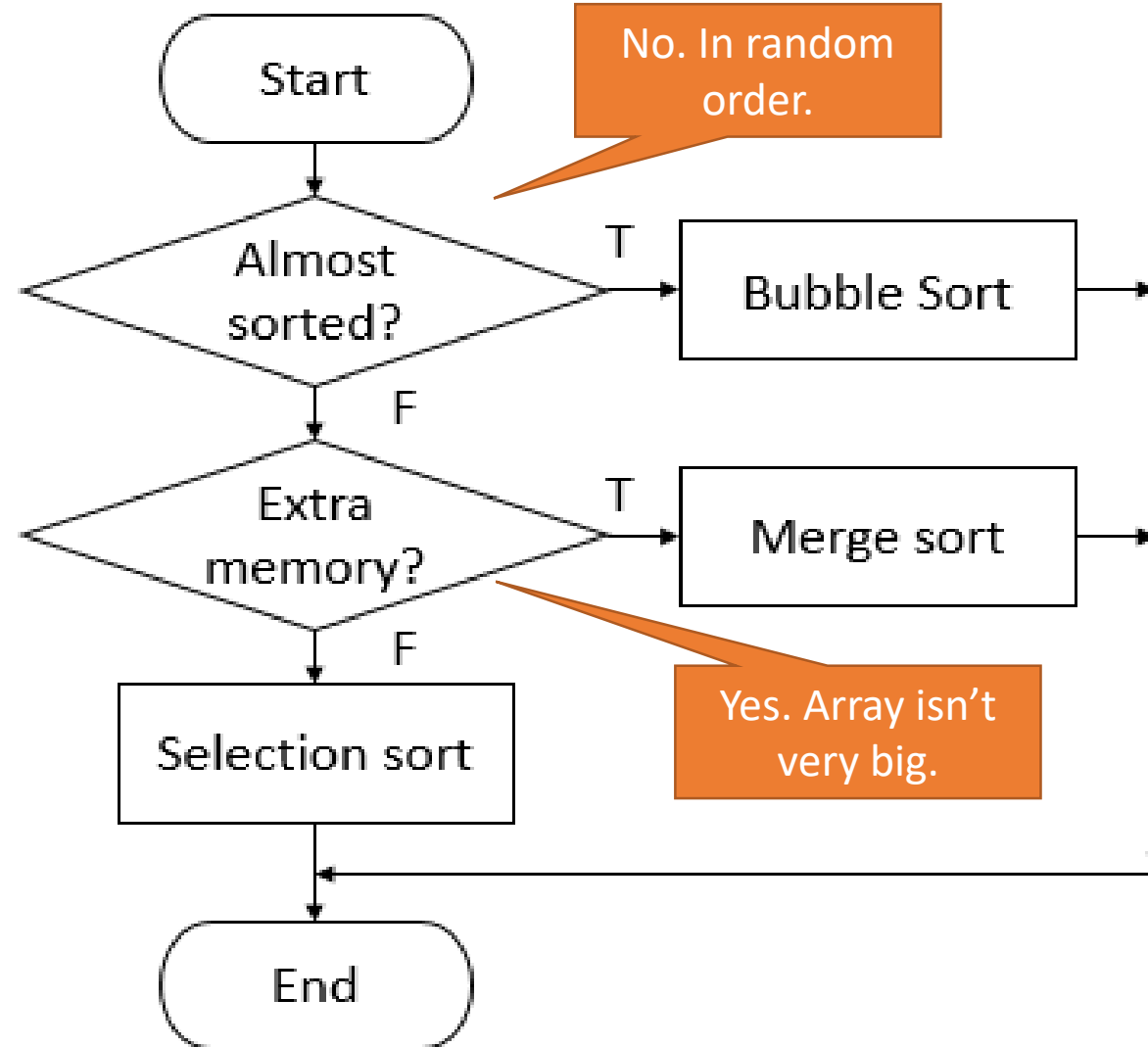
How do you choose which algorithm to use?

The array is 200 elements long (not very big), and it is in completely random order.



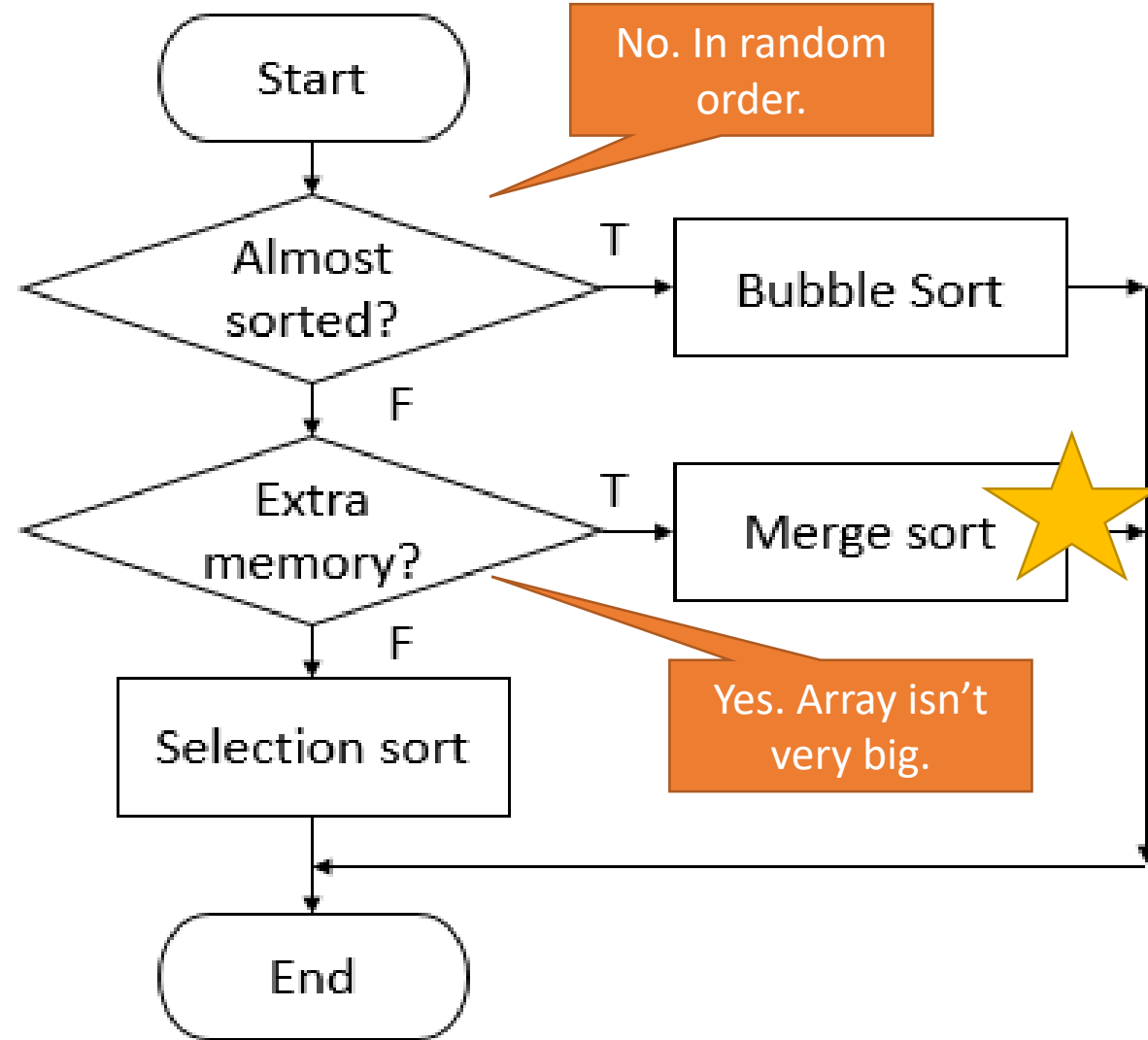
How do you choose which algorithm to use?

The array is 200 elements long (not very big), and it is in completely random order.



How do you choose which algorithm to use?

The array is 200 elements long (not very big), and it is in completely random order.



MERGE SÖRT

idea-instructions.com/merge-sort/
v1.2, CC by-nc-sa 4.0

IDEA

