

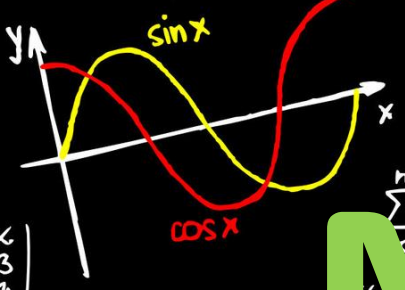
$$3x^2 + y^3 + z^3 + xyz - 6 = 0$$

$$\text{grad} f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

$$\text{tg} x \cdot \text{cotg} x = 1$$

$$2x^2yy' + y^2 = 2$$

$$x_1 = -11p, x_2 = -p, x_3 = 7p, p \in \mathbb{R}$$



$$Y_{i+1} = Y_i + b \cdot k_2$$

$$B = \begin{pmatrix} 2 & 1 & -1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$\text{tg} \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

$$\begin{pmatrix} -k \\ B \\ -\delta \end{pmatrix}$$

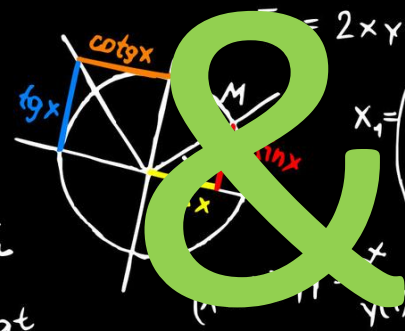
$$\iiint_M z \, dx \, dy \, dz = \int_0^1 \int_0^1 \int_0^1 r \, dr \, d\theta \, dz$$

$$\sum_{i=1}^n (p_2(x_i) - y_i)^2$$

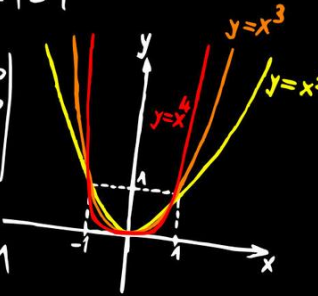
$$\text{tg} 2x = \frac{2 \text{tg} x}{1 - \text{tg}^2 x}$$

$$\text{tg} x = \frac{\sin x}{\cos x}$$

$$\begin{cases} \lambda x - y + z = 1 \\ x + \lambda y + z = 2 \\ x + y + \lambda z = 1 \end{cases}$$



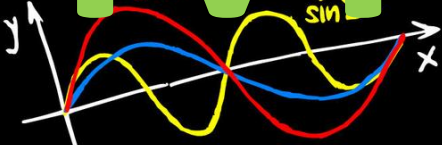
$$2x^2z - 1 = 1$$



# Mod &

$$\arctg x - x = 0, I = (1, 10)$$

$$\int_{\pi/2}^{\pi} \sin^4 x \cdot \cos^3 x \, dx$$



$$\sin^2 \alpha + \cos^2 \beta + \cos^2 \mu = 1$$

$$\delta(p_2) = \sqrt{0,16}$$

$$c = \begin{pmatrix} 0,1 \\ 1,0 \end{pmatrix}$$

$$y = \sqrt[3]{x+1}; x = \text{tg} t$$

$$x_1 = \begin{pmatrix} \alpha + \beta + \gamma \\ \beta \end{pmatrix}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\begin{cases} A+B+C=8 \\ -3A-7B+2C=-10 \\ -18A+6B-3C=1 \end{cases}$$

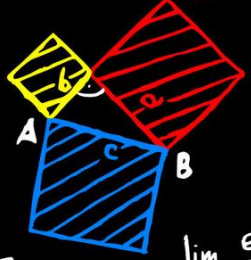
$$z = 2; \frac{\partial z}{\partial y} = 0$$

$$\vec{n} = (F_x; F_y; F_z)$$

$$a^2 + b^2 = c^2$$

# Div

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$



$$f(x) = 2^{-x} + 1, \epsilon = 0,05$$

$$e^{xyz} = A[0; 1]$$

$$\lambda_2 = i\sqrt{14}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\int R(x, \sqrt{\frac{ax+b}{cx+d}}) \, dx$$

$$\frac{\sin x}{x} \leq \frac{x}{x} = 1$$

$$\sin 2x = 2 \sin x \cdot \cos x$$

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{5x} = \frac{2}{5}$$

$$|a| + |b| \neq 0; p \neq 0$$

$$\frac{z}{x} = \frac{1}{x} \arcsin \frac{\sqrt{z}}{2}$$

$$\eta_1 = \lambda_1^2 - 3\lambda_1 + 1 \neq 0$$

$$|z| = \sqrt{a^2 + b^2}$$

$$x \left( \frac{\partial f}{\partial x} \right) = 16 - x^2 + 16y^2 - 4z > 0$$

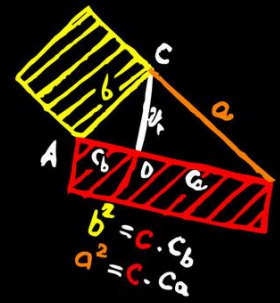
$$A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix}; x=0, y=1, z=2$$

$$y' - \frac{\sqrt{y}}{x+2} = 0; y(0) = 1$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

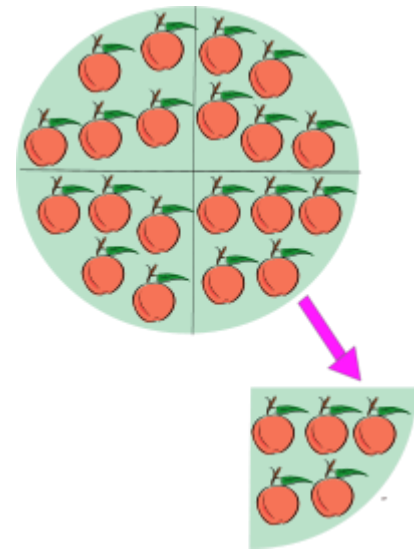
$$A = [1, 0; 3]$$

$$\cos p = \frac{(1, 0) \cdot \left( \frac{1}{2\sqrt{3}}, \frac{1}{4\sqrt{3}} \right)}{\sqrt{\frac{1}{12} + \frac{1}{48}}}$$



With doubles, division works as you'd expect.

$$\text{Eg. } 5/2 = 2.5$$



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With integers, the decimal is **CHOPPED OFF**.

It is **NOT** rounded.

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To get the remainder of the integer division, we use a function called **modulus** or **mod**.

It's symbol is %.

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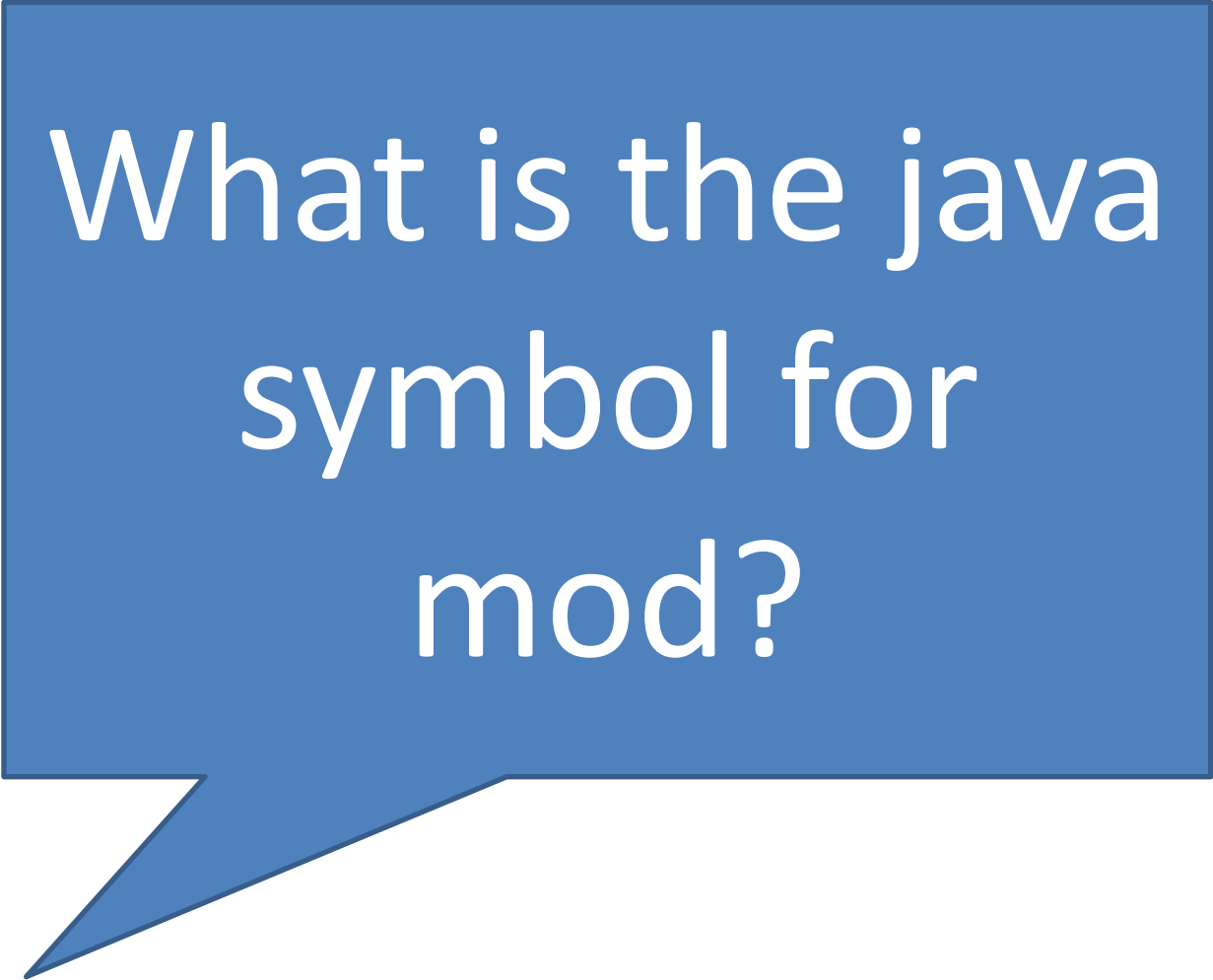
With integers, the decimal is CHOPPED OFF.

It is NOT rounded.

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To get the remainder of the integer division, we use a function called **modulus** or **mod**.

It's symbol is %

A blue speech bubble with a white border and a tail pointing towards the bottom-left. The text inside is white and centered.

What is the java  
symbol for  
mod?

What is the java  
symbol for  
mod?

**%**

A red speech bubble with a white outline and a tail pointing towards the bottom-left. The text inside is white and centered.

Which type uses  
mod?

Which type uses  
mod?

**int**

1. (a) What variable type is used with mod?

(b) What is the mathematical symbol for mod?

(c) How does mod work?

(d) What are two common uses of mod?

(e) How is integer division different?

int

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`determine even/odd`

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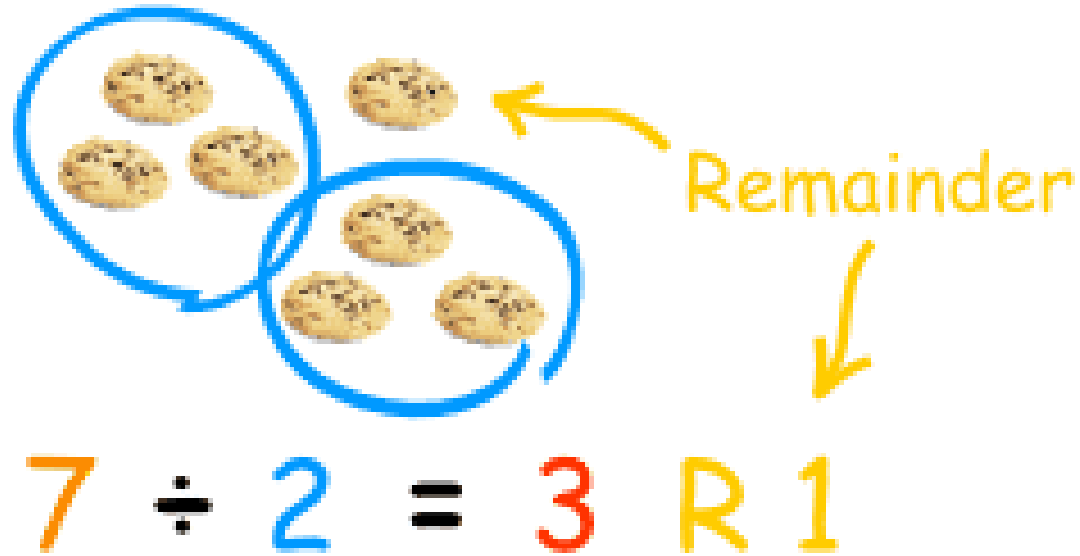
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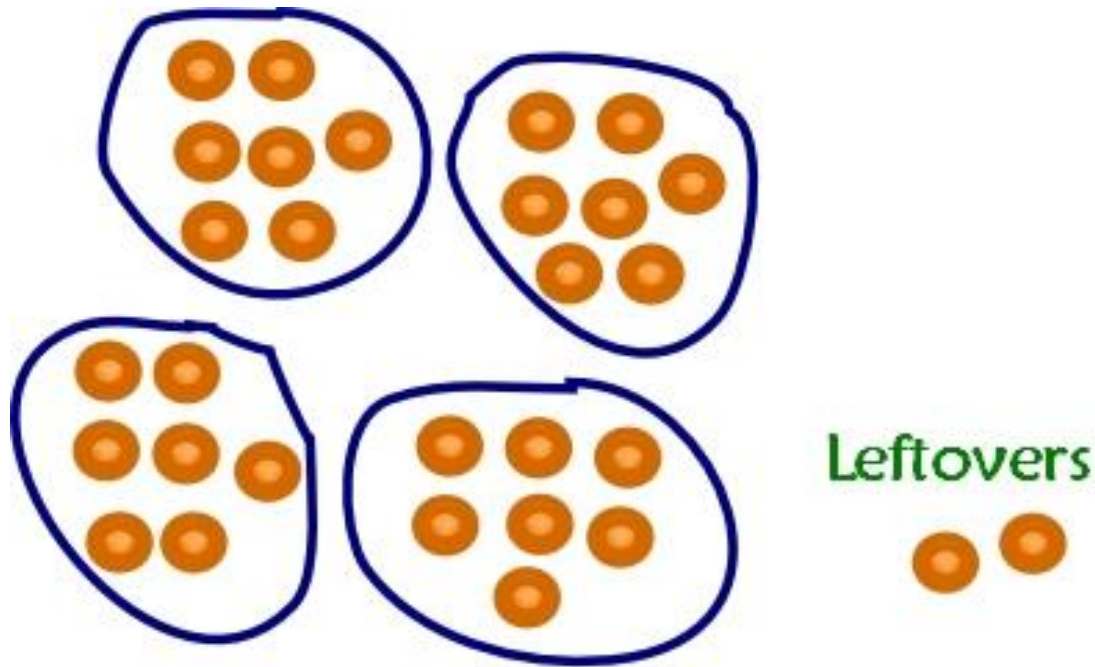
(e) How is integer division different? `do not round`  
`chop off the decimal`



In Java, with integers, we say:

$$7/2 = 3$$

$$7\%2 = 1$$



$$30 \div 7 = 4 \text{ R } 2$$

In Java, with integers, we say:

$$30/7 = 4$$

$$30\%7 = 2$$

Divide 11 into groups of 4: (circle)



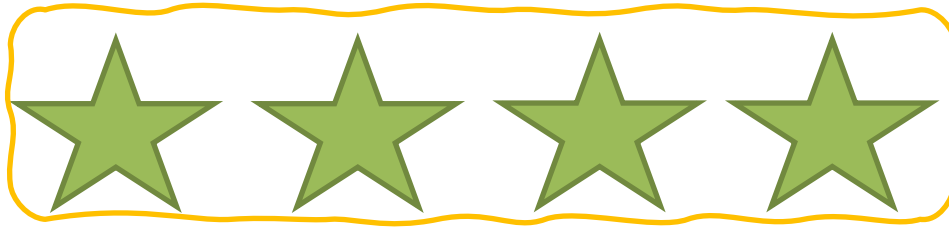
How many circles?

How many are leftover?

What is  $11/4$ ? \_\_\_\_\_

What is  $11\%4$ ? \_\_\_\_\_

Divide 11 into groups of 4: (circle)



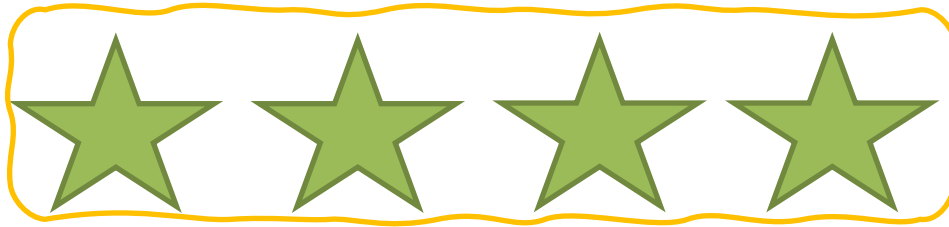
How many circles?

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What is  $11/4$ ? \_\_\_\_\_

What is  $11\%4$ ? \_\_\_\_\_

Divide 11 into groups of 4: (circle)



How many circles?

What is  $11/4$ ? 2

How many are leftover?

What is  $11\%4$ ? 3

Divide 11 into groups of 3: (circle)



How many circles?

How many are leftover?

What is  $11/3$ ? \_\_\_\_\_

What is  $11\%3$ ? \_\_\_\_\_

Divide 11 into groups of 3: (circle)



How many circles?

How many are leftover?

What is  $11/3$ ? \_\_\_\_\_

What is  $11\%3$ ? \_\_\_\_\_

Divide 11 into groups of 3: (circle)

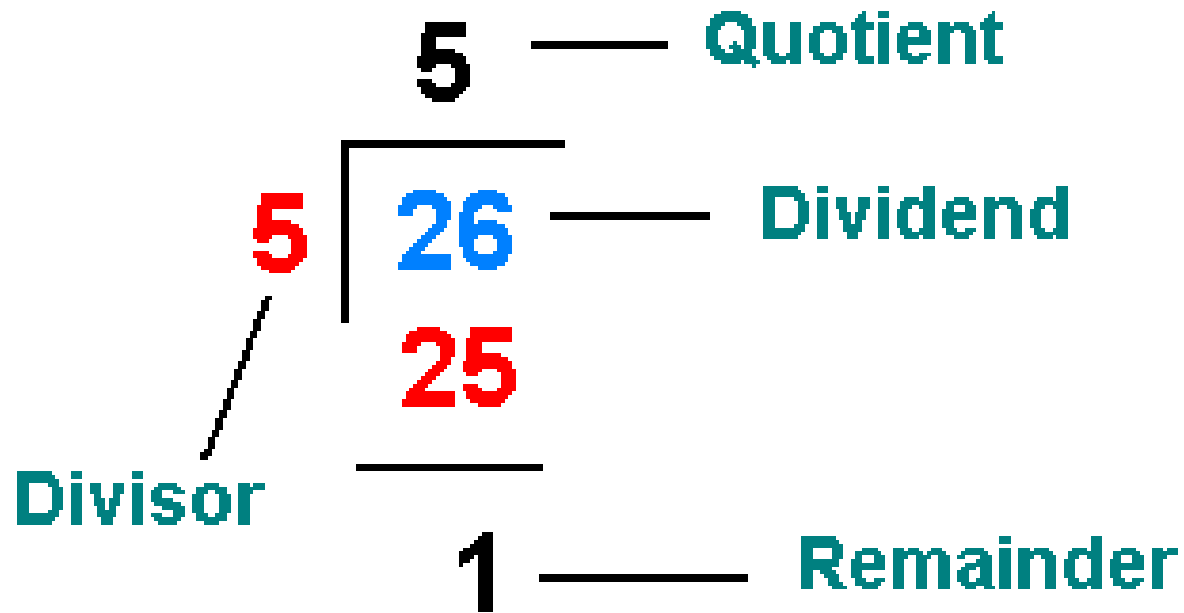


How many circles?

What is  $11/3$ ? 3

How many are leftover?

What is  $11\%3$ ? 2



In Java, with integers, we say:

$$26/5 = 5$$

$$26\%5 = 1$$

Look at the long division and pick out the answers to div and mod.

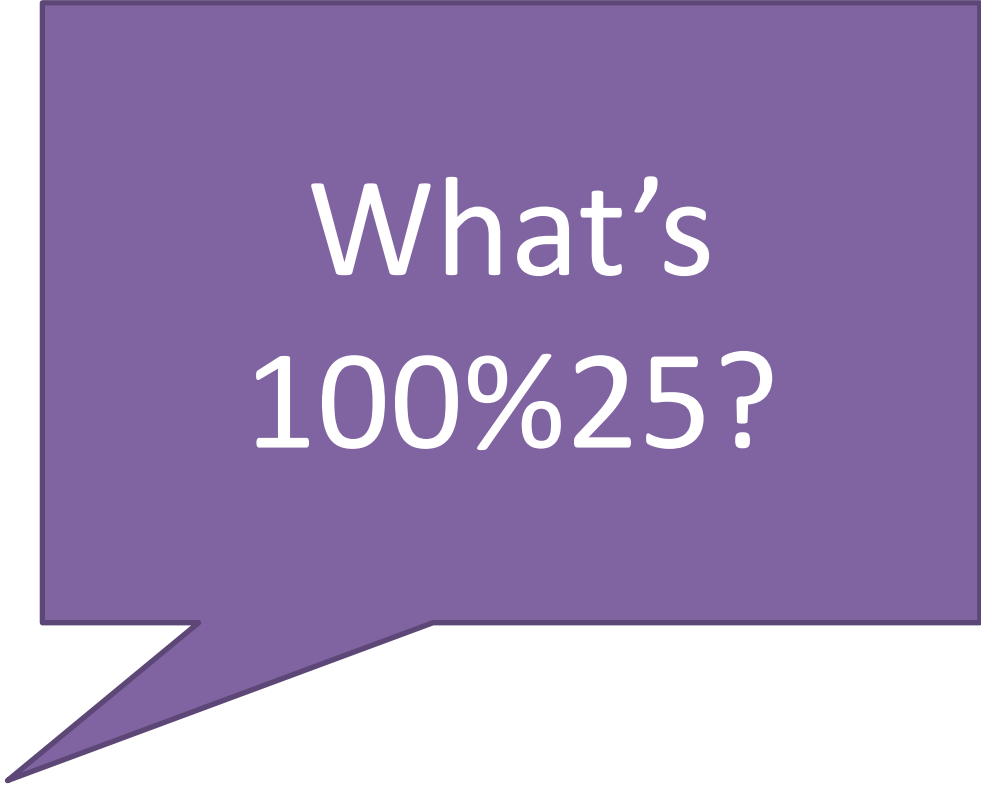
$$\begin{array}{r} 87 \\ \hline 8 \overline{) 703} \\ \underline{-64} \phantom{0} \\ 63 \\ \underline{-56} \\ 7 \end{array}$$

What's the Quotient?

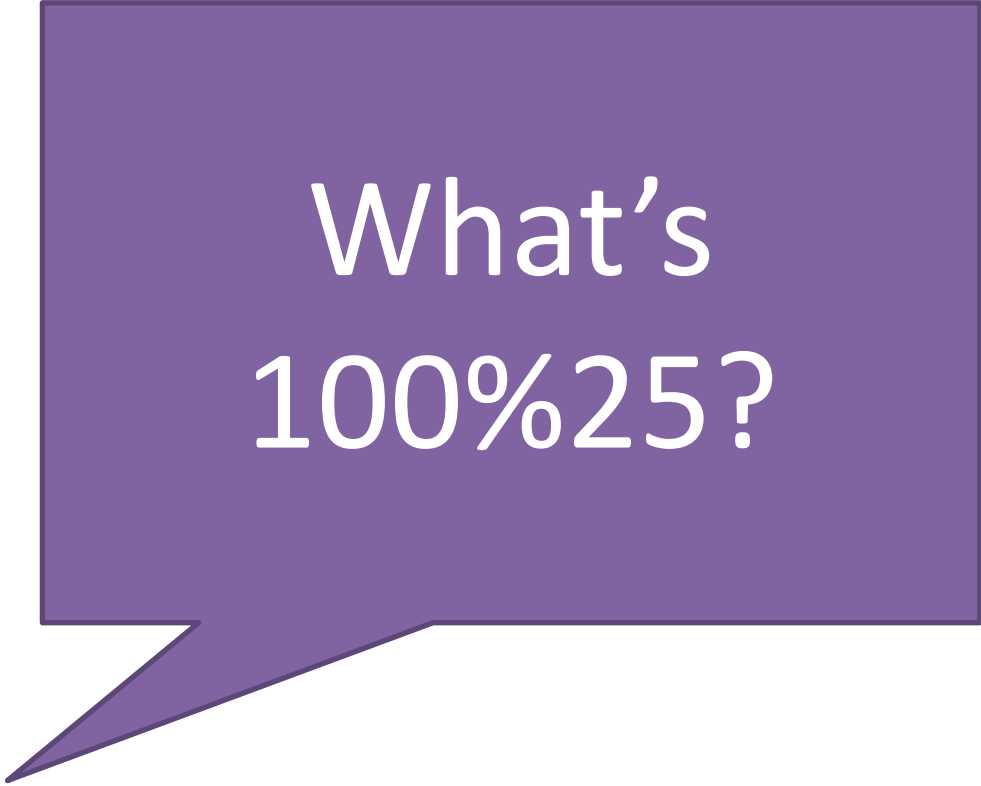
What is  $703/8$ ? \_\_\_\_\_

What's the remainder?

What is  $703\%8$ ? \_\_\_\_\_



What's  
100%25?



What's  
100%25?

0



What's  
12%10?

An orange speech bubble with a white outline and a tail pointing towards the bottom-left. Inside the bubble, the text "What's 12%10?" is written in white.

What's  
12%10?

2




What's  
15%4?



What's  
 $15\%4$ ?

3



What's  
25%6?

What's  
 $25\%6$ ?

1

Let's compare Even and Odd numbers:

Question	Answer	Question	Answer
$8/2$		$8\%2$	
$20/2$		$20\%2$	
$26/2$		$26\%2$	
$3/2$		$3\%2$	
$15/2$		$15\%2$	
$7/2$		$7\%2$	

Let's compare Even and Odd numbers:

Question	Answer	Question	Answer
$8/2$	4	$8\%2$	0
$20/2$	10	$20\%2$	0
$26/2$	13	$26\%2$	0
$3/2$		$3\%2$	
$15/2$		$15\%2$	
$7/2$		$7\%2$	

Let's compare Even and Odd numbers:

Question	Answer	Question	Answer
$8/2$	4	$8\%2$	0
$20/2$	10	$20\%2$	0
$26/2$	13	$26\%2$	0
$3/2$	1	$3\%2$	1
$15/2$	7	$15\%2$	1
$7/2$	3	$7\%2$	1

If x is 14, is this  
Boolean expression  
true or false?

$$x \% 2 == 0$$

If x is 14, is this  
Boolean expression  
true or false?

$$x \% 2 == 0$$

$$14 \% 2 == 0$$

If x is 14, is this  
Boolean expression  
true or false?

$$x \% 2 == 0$$

$$14 \% 2 == 0$$

$$0 == 0$$

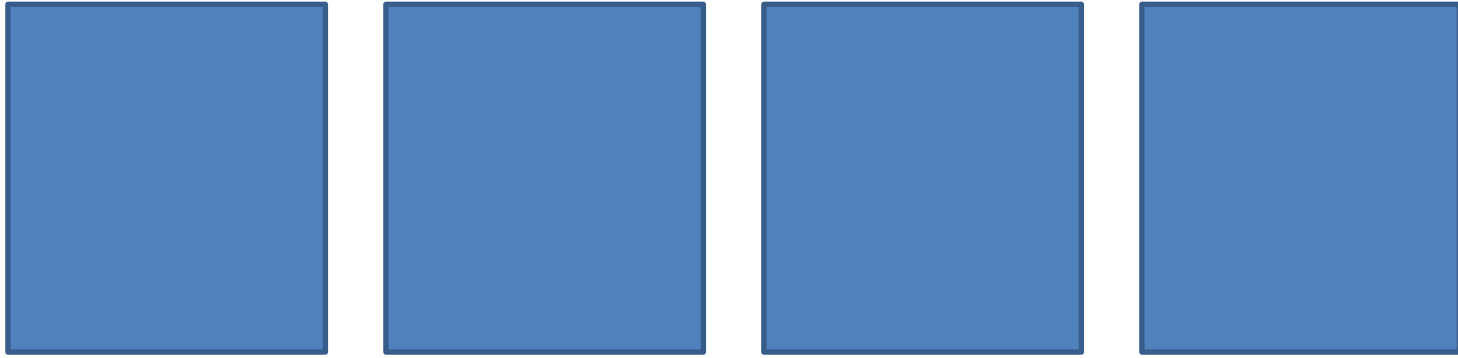
If x is 14, is this  
Boolean expression  
true or false?

$$x \% 2 == 0$$

$$14 \% 2 == 0$$

$$0 == 0$$

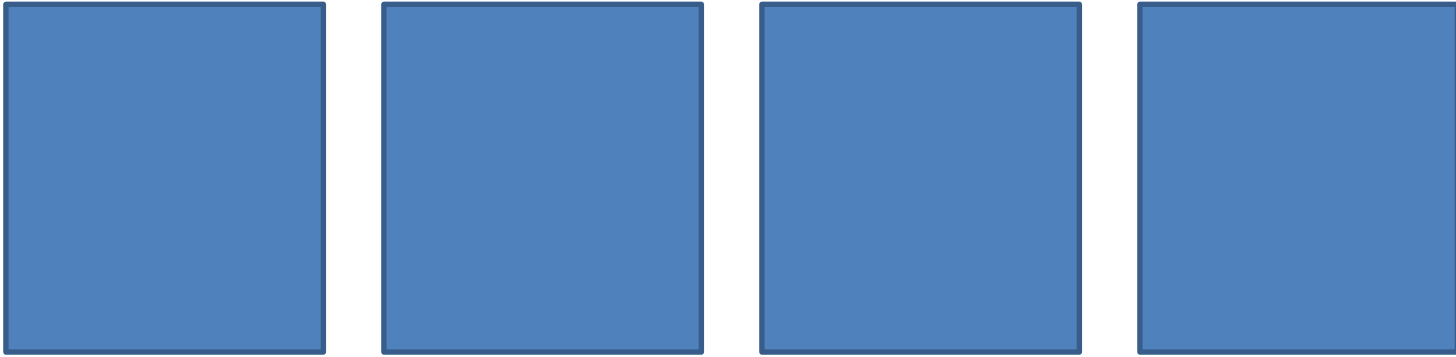
true



What is  $4 / 5$ ?

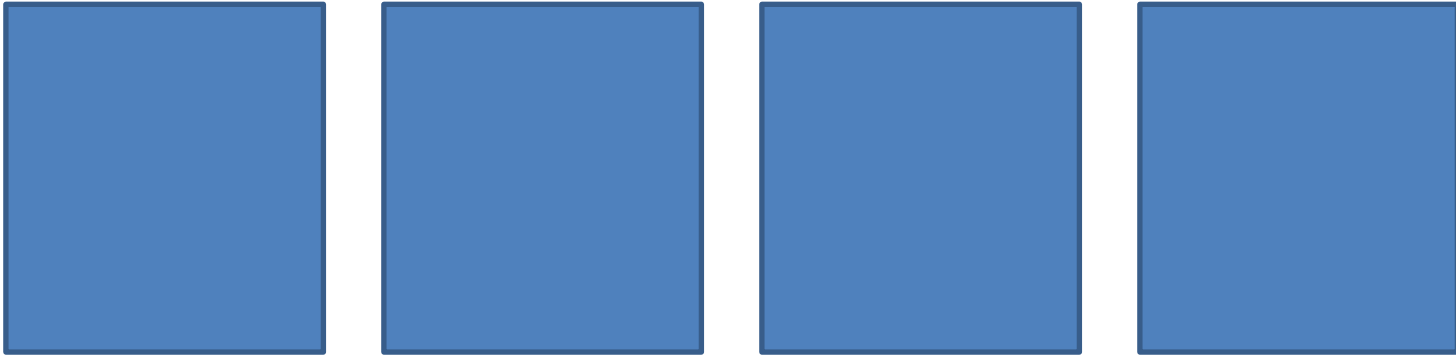
What is  $4 \% 5$ ?

Try these two questions.  
Write in your best guess.



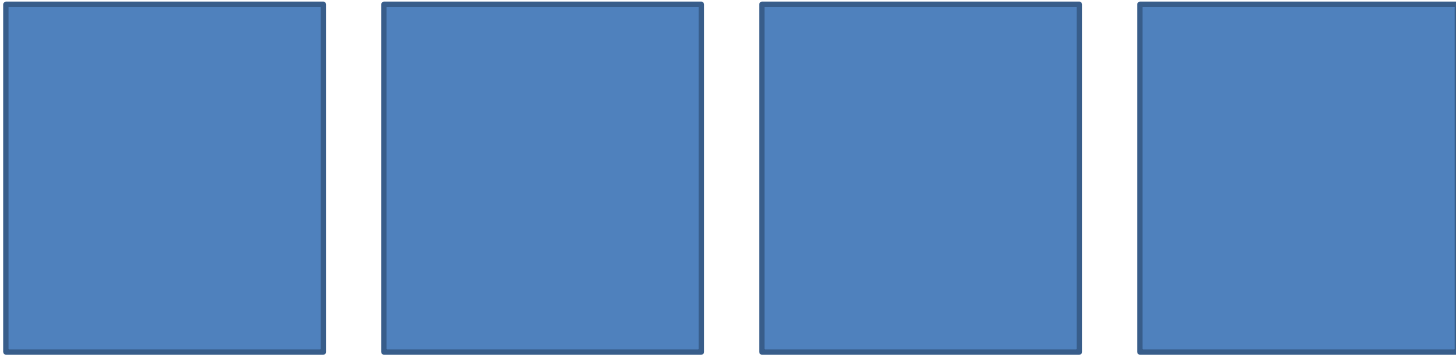
What is  $4 / 5$ ? 0

What is  $4 \% 5$ ?



What is  $4 / 5$ ? 0

What is  $4 \% 5$ ? 4



What is  $4 / 5$ ? 0

What is  $4 \% 5$ ? 4

If the first number is smaller, than all of them remain.

Good old Zero, causing problems again....

What is  $0 / 5$ ?

What is  $0 \% 5$ ?

What is  $5 / 0$ ?

What is  $5 \% 0$ ?

Try these two questions. Write in your best guess.

# Good old Zero, causing problems again....

What is  $0 / 5$ ? 0

What is  $0 \% 5$ ? 0

What is  $5 / 0$ ? undefined

What is  $5 \% 0$ ? undefined













This code mods a number by **5** so that you can see the answer.

```
int n = IO.inputInt("Enter the value: ");  
int answer = n%5;  
System.out.println(n + "%5=" + answer + ", 0 = 5 multiple");
```

- a) What are the names of the two variables?   n   and   answer  .
- b) If the user enters 25 for the number, what is printed?

2	5	%	5	=	0	,		0	=	5		m	u	l	t	i	p	l	e	
---	---	---	---	---	---	---	--	---	---	---	--	---	---	---	---	---	---	---	---	--







This code calculates how many dimes you will get for change.

```
int cents = IO.inputInt("How many cents? ");
int dimes = cents/10;
int leftover = cents%10;
System.out.println("# Dimes: " + dimes);
System.out.println("Remaining change: " +leftover);
```

(a) If the user enters 24 for the number of cents what is printed?

#	D	i	m	e	s	:	2													
R	e	m	a	i	n	i	n	g		c	h	a	n	g	e	:	4			

(b) If the user enters 59 for the number of cents what is printed?

#	D	i	m	e	s	:	5													
R	e	m	a	i	n	i	n	g		c	h	a	n	g	e	:	9			

What's the class name?

```
public class makingChange
{
    public static void main (String args[])
    {
        new makingChange ();
    }
    public makingChange ()
    {
        int cents = IO.inputInt ("How many cents? ");
        int quarters = cents / 25;
        cents = cents % 25;
        int dimes = cents / 10;
        cents = cents % 10;
        int nickles = cents / 5;
        cents = cents % 5;
        System.out.println ("That is " + quarters + " quarters, ");
        System.out.println (dimes + " dimes, " + nickles + " nickles, ");
        System.out.println (cents + " pennies.");
    }
}
```

What's the class name?

makingChange

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public class makingChange
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        System.out.println ("That is " + quarters + " quarters, ");
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```

What are the variable types?


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public class makingChange
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```

What are the variable types?

int

```
public class makingChange
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```

Trace the program.


```
public class makingChange
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    public static void main (String args[])
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    }
}
```

Cents = 94

Trace the program.

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public class makingChange
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    public makingChange ()
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        int cents = IO.inputInt ("How many cents? ");
        int quarters = cents / 25;
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        int dimes = cents / 10;
        cents = cents % 10;
        int nickels = cents / 5;
        cents = cents % 5;
        System.out.println ("That is " + quarters + " quarters, ");
        System.out.println (dimes + " dimes, " + nickels + " nickles, ");
        System.out.println (cents + " pennies.");
    }
}
```

Cents = 94  
Quarters =  $94 / 25 = 3$



Trace the program.

```
public class makingChange
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    public static void main (String args[])
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        System.out.println (dimes + " dimes, " + nickels + " nickles, ");
        System.out.println (cents + " pennies.");
    }
}
```

Cents = 94

Quarters =  $94 / 25 = 3$

Cents =  $94 \% 25 = 19$

Trace the program.

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        System.out.println ("That is " + quarters + " quarters, ");
        System.out.println (dimes + " dimes, " + nickels + " nickles, ");
        System.out.println (cents + " pennies.");
    }
}
```

Cents = 94

Quarters =  $94 / 25 = 3$

Cents =  $94 \% 25 = 19$

Dimes =  $19 / 10 = 1$

Trace the program.

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        System.out.println (cents + " pennies.");
    }
}
```

Cents = 94  
Quarters =  $94 / 25 = 3$   
Cents =  $94 \% 25 = 19$   
Dimes =  $19 / 10 = 1$   
Cents =  $19 \% 10 = 9$

Trace the program.

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public class makingChange
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    }
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Cents = 94  
Quarters =  $94 / 25 = 3$   
Cents =  $94 \% 25 = 19$   
Dimes =  $19 / 10 = 1$   
Cents =  $19 \% 10 = 9$   
Nickels =  $9 / 5 = 1$

Trace the program.

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public class makingChange
{
    public static void main (String args[])
    {
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        int cents = IO.inputInt ("How many cents? ");
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        int dimes = cents / 10;
        cents = cents % 10;
        int nickels = cents / 5;
        cents = cents % 5;
        System.out.println ("That is " + quarters + " quarters, ");
        System.out.println (dimes + " dimes, " + nickels + " nickles, ");
        System.out.println (cents + " pennies.");
    }
}
```

Cents = 94  
Quarters =  $94 / 25 = 3$   
Cents =  $94 \% 25 = 19$   
Dimes =  $19 / 10 = 1$   
Cents =  $19 \% 10 = 9$   
Nickels =  $9 / 5 = 1$   
Cents =  $9 \% 5 = 4$

Trace the program.

```
public class makingChange
{
    public static void main (String args[])
    {
        new makingChange ();
    }
    public makingChange ()
    {
        int cents = IO.inputInt ("How many cents? ");
        int quarters = cents / 25;
        cents = cents % 25;
        int dimes = cents / 10;
        cents = cents % 10;
        int nickels = cents / 5;
        cents = cents % 5;
        System.out.println ("That is " + quarters + " quarters, ");
        System.out.println (dimes + " dimes, " + nickels + " nickles, ");
        System.out.println (cents + " pennies.");
    }
}
```

Cents = 94  
Quarters =  $94 / 25 = 3$   
Cents =  $94 \% 25 = 19$   
Dimes =  $19 / 10 = 1$   
Cents =  $19 \% 10 = 9$   
Nickels =  $9 / 5 = 1$   
Cents =  $9 \% 5 = 4$

How many cents? 94  
That is 3 quarters,  
1 dimes, 1 nickles,  
4 pennies.